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COLLISION AVOIDANCE MANEUVER OPTIMIZATION

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The paper presents a high accuracy fully analytical formulation to compute the miss distance and collision probability of two approaching objects following an impulsive collision avoidance maneuver. The formulation hinges on a linear relation between the applied impulse and the objects relative motion in the b-plane, which allows to formulate the maneuver optimization problem as an eigenvalue problem. The optimization criterion consists of minimizing the maneuver cost in terms of delta-V magnitude in order to either maximize collision miss distance or to minimize Gaussian collision probability. The algorithm, whose accuracy is verified in representative mission scenarios, can be employed for collision avoidance maneuver planning with reduced computational cost when compared to fully numerical algorithms.

INTRODUCTION

The continuous growth of the space objects population in Earth orbit can in principle be stabilized by preventing massive space objects such as satellites and rocket bodies from colliding with each other. While for derelict objects this can only be done by physically removing them from crowded orbital regions (whenever that will be technologically feasible) when at least one of the two objects has the capability to modify its own orbit a collision avoidance maneuver (COLA) is possible and is routinely conducted whenever a predicted conjunction exceeds a collision probability threshold established by the specific satellite operator.

While collision COLA maneuvers involve lower fuel expenditures compared to satellite deorbiting/reorbiting operations they are conducted several time during the lifetime of a satellite and their frequency is expected to increase as the number of space debris increase and as ground-based tracking systems improve. It will therefore be paramount to devise high-fidelity and high-efficiency optimization strategies to be embedded into dedicated maneuver planning software tools (see for instance[1,2]). Typically, these tools perform an optimization analysis to minimize the maneuver cost, measured for example in terms of required delta-V, given a required upper limit for the collision probability with one or more space objects that are predicted to fly closely to the satellite of interest. This optimization process can be very demanding from the computational point of view as in the most general case the orbital motion of the two colliding objects needs to be propagated numerically and starting from a 3-dimensional parameter space for the input variable (direction of the maneuver impulse in space and maneuver location along the orbit).

One fundamental aspect of this complex optimization process is the modeling of the relative dynamics of the two objects. Recent advances in this regard have been made by one of these authors,[3]

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who derived an accurate analytical approximation of the b-plane relative motion of two colliding bodies in Keplerian orbits following a generic impulsive avoidance maneuver. That formulation, valid for a generic collision geometry and arbitrary eccentricity, was employed as a base for an optimization process aimed at maximizing the collision miss distance between the two colliding objects for a given magnitude of available delta-V providing interesting and sometimes counter-intuitive results. Nevertheless, in order to make the formulation applicable to a realistic scenario three additional steps are needed:

1. Consider collision probability, instead of collision miss distance as the objective function of the optimization problem.
2. Generalize the optimization process to include the case of an initially non-zero miss distance vector at close approach.
3. Analyze the influence of environmental perturbations.

The goal of the present work is to tackle these aspects using closed-form analytical expressions and to propose an efficient numerical scheme to solve the optimization problem in its most general form. One crucial advantage of the proposed formulation is the linear dependence between the applied Δv impulse and the displacement along the collision b-plane. This allows to write the objective function (i.e. the collision miss distance or the collision probability) as a quadratic form eventually reducing the optimization problem to the solution of a simple eigenvalue problem (see Conway[4] for a similar result applied to impulsive asteroid deflection) and the solution of a simple non-linear algebraic equation.

The article is organized as follows. First we review the computation of the collision probability between two objects given their relative b-plane position and covariance matrices and following the approach presented in reference.[5] Next we develop our optimization strategy starting from the linear dynamics formulation of reference[3] and addressing the maximum distance and the minimum collision probability problems including the generic case of a predicted b-plane offset before the maneuver. We then apply the proposed method to the 2009 Iridium-Cosmos collision comparing the maximum miss distance with the minimum collision probability scenario. Finally we test the accuracy of the method with a numerical model including the perturbing acceleration of the J2 gravitational harmonic.

COLLISION PROBABILITY

Let us consider two objects S_1 and S_2 experiencing a conjunction event with an expected closest approach relative position $\mathbf{r}_e$. Let us assume that a collision would take place whenever the following condition is verified:

$$\|\mathbf{r}\| = \|\mathbf{r}_1 - \mathbf{r}_2\| < s_A,$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the randomly distributed positions of S_1 and S_2 and s_A can be taken as the sum of the radii of the spherical envelopes centered at S_1 and S_2, respectively.

The probability of collision between S_1 and S_2 can be written, in general terms, as the triple integral of the probability distribution function $f_{\mathbf{r}}(\mathbf{r})$ of the relative position of S_1 with respect to S_2 over the volume V swept by sphere of radius s_A centered at S_2:

$$P = \int_V f_{\mathbf{r}}(\mathbf{r})\, d\mathbf{r}. \tag{1}$$

When the statistical distribution $f_{\mathbf{r}}(\mathbf{r})$ is Gaussian it can be written as:

$$f_{\mathbf{r}}(\mathbf{r}) = \frac{\exp\left(-\frac{1}{2}(\mathbf{r}-\mathbf{r}_e)^T \mathbf{C}_{\mathbf{r}}^{-1}(\mathbf{r}-\mathbf{r}_e)\right)}{(2\pi)^{3/2}\sqrt{\det(\mathbf{C}_{\mathbf{r}})}}, \tag{2}$$

where $\mathbf{C}_{\mathbf{r}}$ is the covariance matrix of $\mathbf{r}$, which corresponds to the sum of the individual covariance matrices of $\mathbf{r}_1$ and $\mathbf{r}_2$, expressed in the same orthonormal base, when the two quantities are statistically independent.

When the temporal extent of the conjunction is small compared to the orbit period of the objects one can consider the motion of the two objects of S_1 and S_2 as *uniform rectilinear* with deterministically known velocities $\mathbf{v}_1$ and $\mathbf{v}_2$, and compute the collision probability as a two-dimensional integral on the collision b-plane.

To this end we define the S_2-centered b-plane reference system $<\xi,\eta,\zeta>$ as in[6] and with:

$$\mathbf{u}_\xi = \frac{\mathbf{v}_2 \times \mathbf{v}_1}{\|\mathbf{v}_2 \times \mathbf{v}_1\|}, \quad \mathbf{u}_\eta = \frac{\mathbf{v}_1 - \mathbf{v}_2}{\|\mathbf{v}_1 - \mathbf{v}_2\|}, \quad \mathbf{u}_\zeta = \mathbf{u}_\xi \times \mathbf{u}_\eta.$$

Under the rectilinear approximation V becomes a cylinder along the η axis and Eq. 1 can now be written in $<\xi,\eta,\zeta>$ axes and integrated for $-\infty < \eta < +\infty$ to yield:

$$P = \int_A \frac{1}{2\pi\sigma_\xi\sigma_\zeta\sqrt{1-\rho_{\xi\zeta}^2}} \exp\left\{-\left[\left(\frac{\xi-\xi_e}{\sigma_\xi}\right)^2 + \left(\frac{\zeta-\zeta_e}{\sigma_\zeta}\right)^2 - 2\rho_{\xi\zeta}\frac{(\zeta-\zeta_e)}{\sigma_\zeta}\frac{(\xi-\xi_e)}{\sigma_\xi}\right] \Big/ 2\left(1-\rho_{\xi\zeta}^2\right)\right\} d\xi d\zeta \tag{3}$$

where $\mathbf{r}_e = (\xi_e, 0, \zeta_e)^T$ is the expected closest approach relative position in b-plane axes, A is a circular domain of radius s_A and σ_ξ, σ_ζ, $\rho_{\xi\zeta}$ can be extracted from the relative position covariance matrix in b-plane axes whose (ξ,ζ) minor reads:

$$\mathbf{C}_{\xi\zeta} = \begin{bmatrix} \sigma_\xi^2 & \rho_{\xi\zeta}\sigma_\xi\sigma_\zeta \\ \rho_{\xi\zeta}\sigma_\xi\sigma_\zeta & \sigma_\zeta^2 \end{bmatrix}.$$

Using Chan's approach (see[5] for details) the computation of Eq (3) can be made equivalent to integrating a properly scaled isotropic Gaussian distribution function over an elliptical cross-section. If the latter is approximated as a circular cross-section of equal area the final computation of the impact probability reduces to a Rician integral that can be computed with the convergent series:

$$P(u,v) = e^{-v/2}\sum_{m=0}^{\infty}\frac{v^m}{2^m m!}\left(1 - e^{-u/2}\sum_{k=0}^{m}\frac{u^k}{2^k k!}\right) \tag{4}$$

with:

$$u = \frac{s_A^2}{\sigma_\xi \sigma_\zeta \sqrt{1-\rho_{\xi\zeta}^2}} \tag{5}$$

$$v = \left(\frac{\xi_e}{\sigma_\xi}\right)^2 + \left(\frac{\zeta_e}{\sigma_\zeta}\right)^2 - 2\rho_{\xi\zeta}\frac{\xi_e}{\sigma_\xi}\frac{\zeta_e}{\sigma_\zeta}. \tag{6}$$

From the above equations it appears that the collision probability is constant when the impact point (ξ_e, ζ_e) belongs to an ellipse of semi-axes ratio σ_ξ/σ_ζ and rotated by an angle:

$$\Theta = \frac{1}{2}\tan^{-1}\left(\frac{2\rho_{\xi\zeta}\sigma_\xi\sigma_\zeta}{\sigma_\xi^2 - \sigma_\zeta^2}\right).$$

In addition, the collision probability decreases exponentially for increasing v, i.e. as the size of the ellipse increases.

MANEUVER OPTIMIZATION

In this section, the optimum direction for an impulsive collision avoidance maneuver for minimizing collision probability is computed. The optimization is based on a linear relation derived in reference[3] between the b-plane impact point displacement and the applied maneuver impulse. After recalling the previous relation we analyze the optimum maneuver maximizing the collision miss-distance before considering the collision probability minimization problem. The two problems are compared. Finally we generalize the optimization for the case of no direct impact ($\xi_e, \zeta_e \neq 0$).

Collision avoidance dynamics

Following reference[3] let us suppose a direct collision ($\xi_e = \zeta_e = 0$) is predicted when the maneuverable satellite S_1 has orbital true anomaly θ_c, radial orbital distance R_c and eccentricity e_0. Let the velocity of S_2 at collision be related to the velocity of S_1 by a (positive) rotation of by an angle $-\pi < \phi < \pi$ around the S_1 orbital plane normal $\mathbf{u}_{\text{h}1}$:

$$\phi = \text{atan2}\left[(\text{v}_1 \times \mathbf{v}_2)\cdot \mathbf{u}_{\text{h}1}, \mathbf{v}_1 \cdot \mathbf{v}_2\right], \tag{7}$$

followed by an out-of-plane rotation $-\pi/2 < \psi < \pi/2$ in the direction approaching $\mathbf{u}_{\text{h}1}$:

$$\psi = \tan^{-1}\left[\frac{(\mathbf{v}_2 \cdot \mathbf{u}_\text{h})\,\|\mathbf{v}_2 \times \mathbf{u}_\text{h}\|}{v_2^2 - (\mathbf{v}_2 \cdot \mathbf{u}_\text{h})^2}\right], \tag{8}$$

and by rescaling its magnitude (v_1) by a factor $\chi = v_2/v_1$.

The resulting b-plane shift $\Delta\mathbf{r} = (\Delta\xi, \Delta\eta, \Delta\zeta)$ after the maneuver impulse $\Delta\mathbf{v} = (\Delta v_r, \Delta v_\theta, \Delta v_h)$ performed at an angular distance $\Delta\theta = \theta_c - \theta_m$ from the expected collision obeys the linear relation:

$$\Delta\mathbf{r} = \mathbf{RKD}\Delta\mathbf{v} = \mathbf{M}\Delta\mathbf{v} \tag{9}$$

where:

$$\mathbf{R} = \begin{bmatrix} 0 & 0 & -1 \\ \cos\beta & -\sin\beta & 0 \\ -\sin\beta & -\cos\beta & 0 \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} -\frac{v_1\sqrt{\mu}}{R_c} & \sin\alpha\sin\theta_c & 0 \\ 0 & -\frac{\cos\alpha\sin\phi\cos\psi}{\sqrt{1-\cos^2\psi\cos^2\phi}} & \frac{\sin\psi}{\sqrt{1-\cos^2\psi\cos^2\phi}} \\ 0 & \frac{\cos\alpha\sin\phi}{\sqrt{1-\cos^2\psi\cos^2\phi}} & \frac{\sin\phi\cos\psi}{\sqrt{1-\cos^2\psi\cos^2\phi}} \end{bmatrix},$$

$$\mathbf{D} = \sqrt{\frac{R_c^3}{\mu}} \begin{bmatrix} c_{tr} & c_{t\theta} & 0 \\ c_{rr} & c_{r\theta} & 0 \\ 0 & 0 & c_{wh} \end{bmatrix}.$$

In the above equations β represents the angle between $\mathbf{v}_1$ and $\mathbf{v}_{rel}$ and obeys:

$$\cos\beta = \frac{1-\chi\cos\psi\cos\phi}{\sqrt{1-2\chi\cos\psi\cos\phi+\chi^2}}, \qquad \sin\beta = \sqrt{1-\cos^2\beta}, \tag{10}$$

μ is the Earth gravitational constant and α is the flight path angle of S_1 at collision, which obeys:

$$\sin\alpha = \frac{e_0\sin\theta_c}{\sqrt{e_0^2+2e_0\cos\theta_c+1}}; \qquad \cos\alpha = \frac{1}{\sqrt{e_0^2+2e_0\cos\theta_c+1}}.$$

Finally the terms c_{tr}, $c_{t\theta}$, c_{rr},$c_{r\theta}$,c_{wh} are non-dimensional functions of e_0, θ_c and θ_m provided in reference.[3] For the the singular case corresponding to $\cos\psi\cos\phi = \pm 1$ the matrix $\mathbf{K}$ yields:

$$\mathbf{K}_0 = \begin{pmatrix} -\frac{v_1\sqrt{\mu}}{R_c} & \sin\alpha\sin\theta_c & 0 \\ 0 & -\cos\alpha & 1 \\ 0 & \cos\alpha & 1 \end{pmatrix}.$$

Maximum miss distance maneuver

The maximum miss distance optimization problem corresponds to finding the direction of the applied impulse $\Delta\mathbf{v}$ impulse, with $|\Delta\mathbf{v}| \le \Delta v_0$, that maximizes the distance of the b-plane intersection of S_1 from the b-plane origin and can be formulated as:

$$\begin{aligned} \text{maximize} \quad & J_r(\Delta\mathbf{v}) = \Delta\xi^2 + \Delta\zeta^2 \\ \text{subject to} \quad & f(\Delta\mathbf{v}) = \Delta\mathbf{v}^T\Delta\mathbf{v} - \Delta v_0^2 \le 0 \end{aligned} \tag{11}$$

J_r can be written as:

$$J_r = \Delta\mathbf{r}^T\mathbf{Q}\Delta\mathbf{r}, \tag{12}$$

with:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{13}$$

and by use of Eq. 9:

$$J_r = \Delta\mathbf{v}^T\mathbf{M}^T\mathbf{Q}\mathbf{M}\Delta\mathbf{v} = \Delta\mathbf{v}^T\mathbf{A}\Delta\mathbf{v}. \tag{14}$$

The optimization problem can now be conveniently solved with the method of Lagrange multipliers. After introducing the Lagrangian function:

$$L\left(\Delta\mathbf{v}, \lambda\right) = J_r - \lambda f \tag{15}$$

the necessary conditions for the existence of a maximum obey:

$$\frac{\partial L}{\partial \Delta\mathbf{v}} = 2\mathbf{A}\Delta\mathbf{v} - 2\lambda\Delta\mathbf{v} = 0, \tag{16}$$

which is an eigenvalue problem. After a quick inspection it can be shown that:

$$\text{rank}\left(\mathbf{A}\right) = \begin{cases} 1 & \text{for } \theta_c - \theta_m = 2\pi n \\ 2 & \text{otherwise} \end{cases},$$

meaning that there is always at least one impulse direction leaving the collision unaffected. When two non-zero eigenvalues are present the optimal solution is associated with the maximum eigenvalue λ_1 with the corresponding unit eigenvector $\mathbf{s}_1$ providing the direction of the optimal impulse:

$$\Delta\mathbf{v}_{opt} = \Delta v_0 \mathbf{s}_1$$

and the corresponding maximum miss distance:

$$\Delta r_{max} = \sqrt{\lambda_1}\Delta v_0.$$

Note that for the direct-hit case analyzed here the maximum distance can be obtained with both a positive and negative Δv_0 impulse along the $\mathbf{s}_1$ direction.

Minimum collision probability maneuver

The minimum collision probability optimization problem can be formulated as:

$$\begin{aligned} &\text{maximize} \quad J_P\left(\Delta\mathbf{v}\right) = \left(\frac{\Delta\xi}{\sigma_\xi}\right)^2 + \left(\frac{\Delta\zeta}{\sigma_\zeta}\right)^2 + 2\rho_{\xi\zeta}\frac{\Delta\xi}{\sigma_\xi}\frac{\Delta\zeta}{\sigma_\zeta} \\ &\text{subject to} \quad f\left(\Delta\mathbf{v}\right) = \Delta\mathbf{v}^T\Delta\mathbf{v} - \Delta v_0^2 \leq 0 \end{aligned} \tag{17}$$

J_δ can be written as:

$$J_P = \Delta\mathbf{r}^T\mathbf{Q}^*\Delta\mathbf{r},$$

with:

$$\mathbf{Q}^* = \begin{bmatrix} \frac{1}{\sigma_\xi^2} & 0 & \frac{\rho_{\xi\zeta}}{\sigma_\xi\sigma_\zeta} \\ 0 & 0 & 0 \\ \frac{\rho_{\xi\zeta}}{\sigma_\xi\sigma_\zeta} & 0 & \frac{1}{\sigma_\zeta^2} \end{bmatrix}.$$

and by use of Eq. 9:

$$J_P = \Delta\mathbf{v}^T\mathbf{M}^T\mathbf{Q}^*\mathbf{M}\Delta\mathbf{v} = \Delta\mathbf{v}^T\mathbf{A}^*\Delta\mathbf{v}.$$

Similarly to the previous case the optimization problem reduces to calculate the eigenvalues and eigenvectors of $\mathbf{A}^*$. The eigenvector $\mathbf{s}_1^*$ associated to the maximum eigenvalue λ_1^* provides the direction of the optimal impulse for minimum collision probability:

$$\Delta\mathbf{v}_{opt}^* = \Delta v_0\mathbf{s}_1^*,$$

and the corresponding minimum collision probability can be computed by substituting:

$$v_{max} = \sqrt{\lambda_1^*}\Delta v_0$$

into Eq. 4.

Non-direct impact

In this section the general case of a non-direct impact is analyzed. Since it has been shown that the maximum miss distance and minimum collision probability are formally equivalent, we will refer to the first case.

When the impact is non-direct ($\mathbf{r}_e \neq \mathbf{0}$) Eq. 9 is generalized to:

$$\Delta\mathbf{r} = \mathbf{r}_e + \mathbf{M}\Delta\mathbf{v} \tag{18}$$

The corresponding squared miss distance (Eqs.(12,13,14)) becomes:

$$\begin{aligned} J_r &= (\mathbf{r}_e + \mathbf{M}\Delta\mathbf{v})^T\,\mathbf{Q}\,(\mathbf{r}_e + \mathbf{M}\Delta\mathbf{v}) \\ &= \mathbf{r}_e^T\mathbf{Q}\mathbf{r}_e + \Delta\mathbf{v}^T\mathbf{A}\Delta\mathbf{v} + 2\mathbf{r}_e^T\mathbf{Q}\mathbf{M}\Delta\mathbf{v}. \end{aligned} \tag{19}$$

After dropping the constant term $\mathbf{r}_e^T\mathbf{Q}\mathbf{r}_e$ and multiplying by Δv_0 the objective function the optimization problem can be conveniently rewritten as:

$$\begin{array}{ll} \text{maximize} & \tilde{J}_r(\mathbf{u}) = \mathbf{u}^T\mathbf{A}\mathbf{u} + 2\mathbf{b}^T\mathbf{u} \\ \text{subject to} & f(\mathbf{u}) = \mathbf{u}^T\mathbf{u} - 1 \leq 0 \end{array}, \tag{20}$$

where we set:

$$\mathbf{u} = \Delta\mathbf{v}/\Delta v_0,$$

$$\mathbf{b}^T = \mathbf{r}_e^T \mathbf{Q}\mathbf{M}/\Delta v_0.$$

The problem is a *non-convex* quadratic optimization problem , which can be reduced to the following *convex* problem:[7]

$$\begin{aligned} \text{minimize} \quad & \frac{\left(\mathbf{s}_1^T\mathbf{b}\right)^2}{\lambda - \lambda_1} + \frac{\left(\mathbf{s}_2^T\mathbf{b}\right)^2}{\lambda - \lambda_2} + \lambda, \\ \text{subject to} \quad & \lambda \geq \lambda_1 \end{aligned} \tag{21}$$

where λ_1, λ_2 and $\mathbf{s}_1, \mathbf{s}_2$ are the two non-zero eigenvalues of $\mathbf{A}$, in descending order, and the corresponding eigenvectors, respectively.

Eq. (21) leads to the condition:

$$\begin{cases} \left(\dfrac{\mathbf{s}_1^T\mathbf{b}}{\lambda - \lambda_1}\right)^2 + \left(\dfrac{\mathbf{s}_2^T\mathbf{b}}{\lambda - \lambda_2}\right)^2 - 1 = 0 \\ \lambda \geq \lambda_1 \end{cases}, \tag{22}$$

which can be easily solved with Newton's method providing λ_{opt}.

Once λ has been determined the corresponding $\Delta\mathbf{v}$ can be obtained as:[7]

$$\Delta\mathbf{v}_{opt} = -\Delta v_0 \left(\mathbf{A} - \lambda_{opt}\mathbf{I}\right)^\dagger \mathbf{b}, \tag{23}$$

where the dagger sign represents the pseudo-inversion matrix operation. The maximum miss distance is finally obtained by substituting Eq.(23) into Eq.(19).

NUMERICAL CASE

To illustrate the results of our optimization we consider a hypothetical collision avoidance maneuver prior to the 2009 Cosmos-Iridium collision. The encounter geometry is summarized in Table 1. In addition we assume an 4 m spherical envelope for the Cosmos 2251 (due to its 6.5 m boom) and a 3 m envelope for Iridium 33 providing $s_A = 7$ m. We then assume, for both objects, a diagonal covariance matrix with standard deviation of 1 km in the along-track and 100m in both the normal and cross track directions*. Employing the formulas of the Appendix for the covariance matrices summation we obtain:

$$\mathbf{C}_{\xi\zeta} = \begin{bmatrix} 0.02\,\text{km}^2 & 0 \\ 0 & 0.80\,\text{km}^2 \end{bmatrix}.$$

Finally, we assume that a non-direct impact is expected with ξ_e =0 and ζ_e =100 m.

A 10 cm/s Δv maneuver is applied at an angular distance $\Delta\theta$ from the conjunction location. The radial, transversal and out-of-plane components of the impulse can be conveniently written as:

*These values are freely chosen as a numerical example and do not represent the actual situation at the time of the collision

Table 1. **Iridium-Cosmos encounter geometry. Here Iridium is the maneuverable satellite (S_1).**

a_0(km)	e_0	ϕ(deg)	ψ(deg)	θ_c(deg)	χ
7155.8	2×10^{-4}	180.0	77.5	-16.85	1.0

$$\Delta v_r = \Delta \mathrm{v} \cos\gamma \sin(\sigma + \alpha)$$

$$\Delta v_\theta = \Delta v \cos\gamma \cos(\sigma + \alpha)$$

$$\Delta v_h = \Delta \mathrm{v} \sin\gamma$$

where α is the flight path angle, σ is the in-plane rotation, opposite to the orbit angular momentum direction, of the maneuver velocity vector with respect to *tangent to the orbit*, and γ is the subsequent rotation along the out-of-plane direction (see[3]). A fully tangential impulse corresponds to $\sigma = \gamma = 0$.

Comparison between minimum-collision-probability and maximum-miss-distance maneuver

Figures (1-3) compare the optimal maneuver orientation angles and the b-plane trajectory for varying $\Delta\theta$ in the case of minimum collision probability and maximum miss distance. Both the angles and the b-plane trajectory look quite different when $\Delta\theta$ is small while they tend to converge to the same value as $\Delta\theta \to \infty$. The difference for small $\Delta\theta$ are due to the fact that the b-plane relative position ellipse is very elongated in the direction of the ζ axis so that a maneuver strategy minimizing collision probability tends to shift the b-plane position towards the ellipse edge rather than to get the farthest away from the center of the b-plane, as it occurs when miss distance is maximized. Note that in the limit case in which $\sigma_\zeta \to \sigma_\xi$ the ellipse would become a circle and the two optimization problems would become equivalent.

Notably, there appear to be deep local minima in the collision probability curve which are not so pronounced in the miss distance one. This suggests that when collision probability has to be minimized the satellite operator should perform the maneuver near very specific "favorable" orbital position to get the maximum benefit. Furthermore there appear to be no relation between the collision miss distance local maxima and the collision probability local minima.

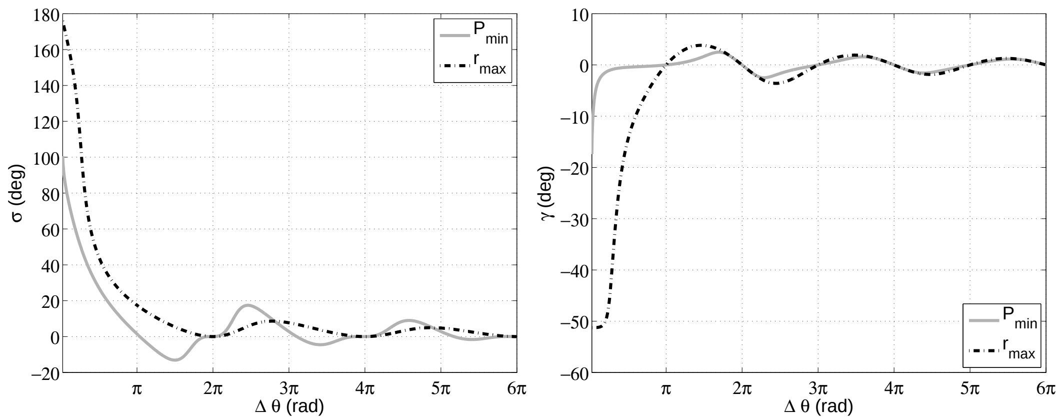

Figure 1. Optimal in-plane (left) and out-of-plane (right) maneuver orientation angles for the Cosmos-Iridium 10 cm/s COLA maneuver.

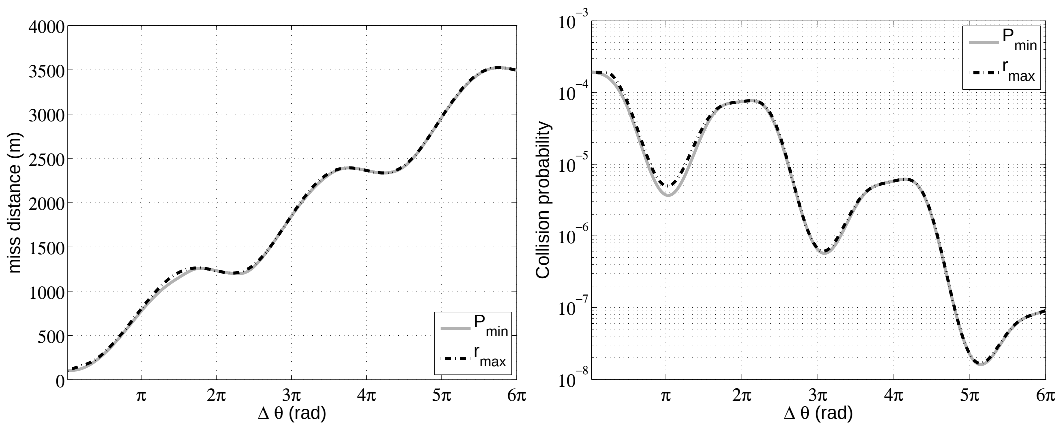

Figure 2. Miss distance (left) and collision probability (right) comparison for the Cosmos-Iridium 10 cm/s COLA maneuver.

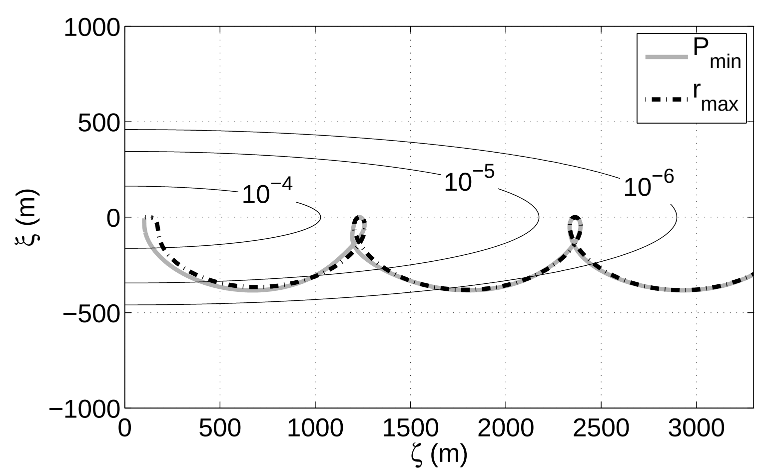

Figure 3. Comparison of b-plane crossing point following a 10 cm/s COLA maneuver using minimum probability and maximum miss distance optimization criteria. Collision probability contour lines are drawn for reference.

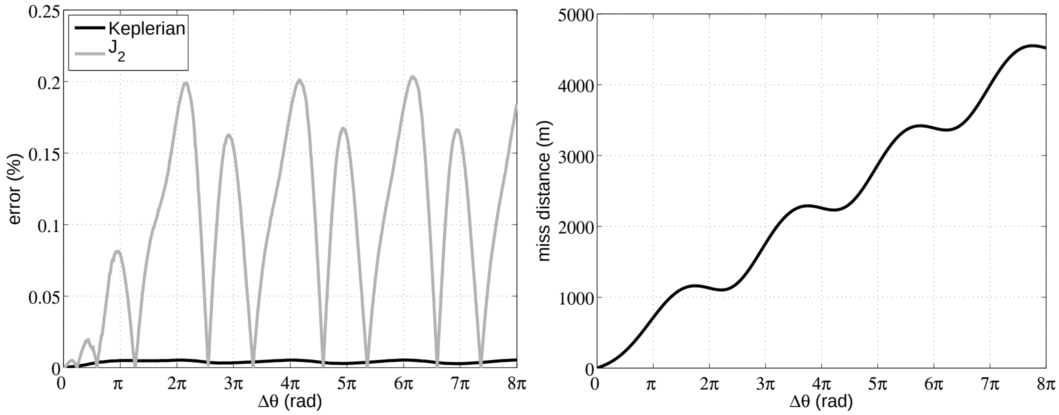

Figure 4. Error of the proposed analytical formulation when compared to a high accuracy analytical propagation for the Cosmos-Iridium 10 cm/s COLA maneuver. The collision miss distance is shown on the right for reference.

ACCURACY OF THE METHOD

As mentioned earlier, the proposed formulation considers Keplerian orbits and neglects environmental perturbations. The approach is justified by the fact that the COLA maneuver causes a relatively small deviation (relative to its orbital distance) of the maneuvered satellite path compared to its original trajectory. The effect of any perturbing acceleration is proportional to such displacement and scales as the acceleration gradient which is orders of magnitudes smaller than the main gravity gradient even for the dominant perturbation in the densely populated part of the LEO environment: the J2 effect.

The error associated to the J2 effect has been investigated numerically and compared to small intrinsic error of the analytical model already proposed in Ref.[3] To this end each orbit has been propagated backward from the conjunction orbital state (at θ_c) up to the maneuver point (θ_m) including the J2 perturbation and then propagated forward with after applying the optimized $\Delta\mathbf{v}$ impulse (again with the J2 perturbation active) up to the collision point to finally compute the numerically accurate miss distance.

Figures (4,5) show the error associated with a J2-perturbed and a Keplerian numerically propagated orbit for the Iridium-Cosmos collision case and for a near head-on collision. An inclination of 86 degrees has been employed for the maneuverable satellite (Iridium) of the former case. The near head-on collision differs from the former case by the angle ψ, set to 1 degree, and by the inclination of the maneuverable satellite (45 degrees).

While the Iridium-Cosmos collision shows negligible error (<0.25%) the near-head-on collision exhibit error peaks reaching almost 9% error. However, the peaks coincides with badly planned maneuvers leading to low achievable miss distance (see Fig (5) right).

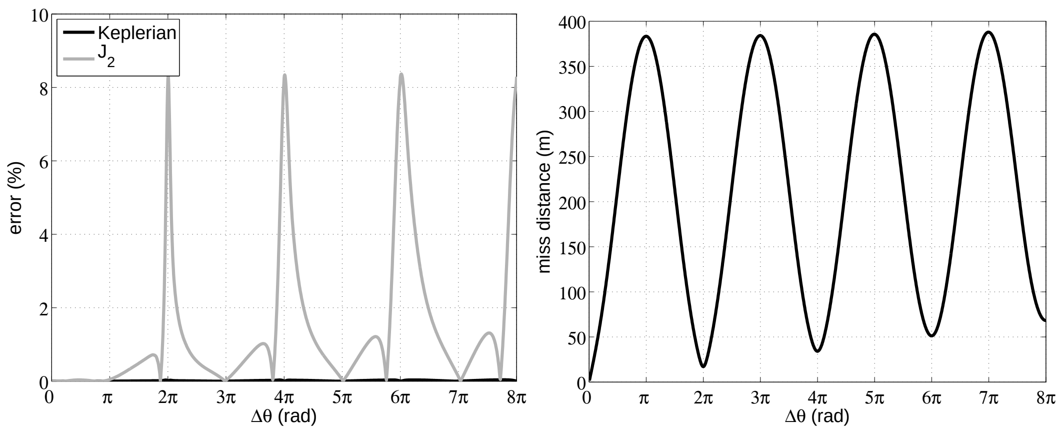

Figure 5. Error of the proposed analytical formulation when compared to a high accuracy analytical propagation for a near head-on 10 cm/s COLA maneuver. The collision miss distance is shown on the right for reference.

CONCLUSIONS

An accurate analytical formulation for the optimization of impulsive collision avoidance maneuvers in low Earth orbit (LEO) has been presented. The use of a previously developed linear formulation linking the b-plane collision point displacement to the applied Δv is shown to be crucial in order to reduce the optimization problem, for the most general case, to the solution of a simple eigenvalue problem and a non-linear algebraic equation. The proposed algorithm works for both maximum miss distance and minimum collision probability optimization and for generic orbital elements and collision geometry. In addition, it has been verified that the most relevant environmental perturbation (the J2 Earth gravitational harmonic) negligibly affects the accuracy of the method. Numerical examples reveal that when the maneuver is conducted less than a few orbits in advance the optimal impulse direction is far from tangential and there is a considerable difference between the minimum collision probability and the maximum miss distance case.

APPENDIX

0.1 Computation of the relative position covariance matrix in b-plane axes

The rotation matrix from S_2 to S_1 Frenet axes is obtained as:

$$\mathbf{R}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\nu & \sin\nu \\ 0 & -\sin\nu & \cos\nu \end{bmatrix} \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{24}$$

where ϕ and ψ can be computed through Eqs. (7,8) while ν can be obtained with the following relation

$$\nu = \text{atan2}\left[(\text{v}_2 \times \mathbf{u}_{\text{h}1}) \cdot \mathbf{u}_{\text{h}2}, v_2 \mathbf{u}_{\text{h}1} \cdot \mathbf{u}_{\text{h}2}\right]. \tag{25}$$

The rotation matrix from S_1 Frenet axes to b-plane axes reads:

$$\mathbf{R}_{1b} = \begin{bmatrix} \left[\mathbf{u}_\xi^T\right]_1 \\ \left[\mathbf{u}_\eta^T\right]_1 \\ \left[\mathbf{u}_\zeta^T\right]_1 \end{bmatrix} \tag{26}$$

where $[\mathbf{u}]_1$ indicates the matrix representation of the unit vector $\mathbf{u}$ in S_1 Frenet axes.

Under the (conservative) assumption that S_1 and S_2 positions are statistically independent the relative position covariance matrix projected onto b-plane axes can be conveniently computed by summing up the individual covariance matrices expressed in S_1 Frenet axes (tangential, normal, out-of-plane) and transforming the resulting matrix into b-plane axes. In this fashion the b-plane relative position covariance matrix reads:

$$\mathbf{C}_b = \mathbf{R}_{1b}\left(\mathbf{C}_1 + \mathbf{R}_{21}\mathbf{C}_2\mathbf{R}_{21}^T\right)\mathbf{R}_{1b}^T,$$

where $\mathbf{C}_1$ and $\mathbf{C}_2$ are, respectively, the covariance matrices of the orbital positions of S_1 and S_2 projected onto the respective Frenet reference systems.

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REFERENCES

[1] S. Alfano, “Collision avoidance maneuver planning tool,” *15th AAS/AIAA Astrodynamics Specialist Conference*, 2005, pp. 7–11.

[2] N. Sanchez-Ortiz, I. Grande-Olalla, J. A. Pulido, and K. Merz, “Collision Risk Assessment and Avoidance Manoeuvres - The new CORAM tool for ESA,” *64 th International Astronautical Congress, Beijing, China*, 2013.

[3] Bombardelli, “Analytical Formulation of Impulsive Collision Avoidance Dynamics,” *Celestial Mechanics and Dynamical Astronomy(in press)*, 2013.

[4] B. A. Conway, “Near-optimal deflection of Earth-approaching asteroids,” *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 5, 2001, pp. 1035–1037.

[5] F. K. Chan, *Spacecraft collision probability*. Aerospace Press, 2008.

[6] G. Valsecchi, A. Milani, G. Gronchi, and S. Chesley, “Resonant returns to close approaches: Analytical theory,” *Astronomy and Astrophysics*, Vol. 408, No. 3, 2003, pp. 1179–1196.

[7] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.