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INVESTIGATING THE SUITABILITY OF ANALYTICAL AND SEMI-ANALYTICAL SATELLITE THEORIES FOR SPACE OBJECT CATALOGUE MAINTENANCE IN GEOSYNCHRONOUS REGIME

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This paper evaluates the performance of two analytical and one semi-analytical orbit propagation theories for artificial Earth satellites in GEO orbital regime. Computationally efficient propagators are required to maintain a catalogue of space objects, i.e. to determine the orbits of the tracked objects, propagating, and correlating them. Studied theories included are, the Simplified General perturbation theories for deep space(SDP4), Kamel's theory for geostationary objects and the Draper Semi-analytical Satellite Theory (DSST). To test the accuracies of the selected propagators, trajectories are compared with a numerical "truth" trajectory. Computational time and RMS errors are used as comparison metrics.

INTRODUCTION

The purpose of this study is to find an optimal (with respect to accuracy and computation time) orbit propagation method which is suitable for space object cataloguing. After the theoretical study of various General Perturbations (GP) and Semi-analytical techniques, the most suited ones are chosen and their performance in terms of accuracy and computational efficiency is compared.

Special Perturbations / Numerical (SP) theories have been studied quite intensively and benchmarked, hence this study does not include them. The selected GP and Semi-analytical theories are used to generate reference orbits; these reference orbits are compared with numerically propagated (Cowell) orbits with physically similar initial conditions. The evaluated theories in this study are GP which includes the Simplified General Perturbation for Deep Space (SDP4) and Kamel's Theory, along with the Semi-analytical technique which includes the Draper Semi-analytical Satellite theory (DSST). Reasons for selection and a brief description of these theories are provided in the theory section. Propagators that are not sufficiently documented or not freely available are not considered in this work.

Semi-analytical and analytical theories are different in the way the forces are modelled. In order to handle low eccentricities and inclinations, the analytical or semi-analytical satellite theory has to be derived using a non-singular orbital element set. High order perturbation modelling is required

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Orbital classes in GEO:

Because different perturbations dominate on different orbits and multiple satellite theories may be employed in tracking/scheduling in Space Situational Awareness (SSA), it was useful to sub-divide the catalogue entries. From the information provided from NORAD Two-Line Elements (TLE) and ESA's DISCOS catalogue, Flohrer[1] classified the objects in the geosynchronous regime and defined the GEO class with orbital elements which are within the following limits:

- Mean motion between 0.9 to 1.1 revolution per sidereal day $(0.9 \leq n \leq 1.1)$,
- Eccentricity smaller than 0.2 $(0 \leq e \leq 0.2)$,
- Inclination smaller than $30^\circ (0^\circ \leq i \leq 30^\circ)$.

Based on this definition, three subclasses of GEO orbits were formulated: uGEO - usual operational range of geostationary orbits, eGEO - eccentric GEO, and iGEO - inclined GEO or geosynchronous orbits. Table 1 gives the definition of these orbital classes. It also shows the number of objects in the specified orbital subclass that are present in the publicly available satellite database `CelesTrak`* released on 30 June 2013.

Table 1. Geo-synchronous Orbital classes considered for analysis

Class	n	i	e	No. of Objects
uGEO	$0.9 \leq n \leq 1.1$	$0^\circ \leq i \leq 5^\circ$	$0 \leq e \leq 0.02$	525
eGEO	$0.9 \leq n \leq 1.1$	$0^\circ \leq i \leq 5^\circ$	$0.02 \leq e \leq 0.2$	120
iGEO	$0.9 \leq n \leq 1.1$	$5^\circ \leq i \leq 30^\circ$	$0 \leq e \leq 0.02$	312

The selected orbit propagation methods are tested extensively on these orbital classes in order to make a comprehensive assessment of each theory's performance. The theories performances are evaluated for their accuracy of orbit prediction. At this point we concentrate on GEO orbits; in a follow up study, we will cover the low and medium earth orbital regimes along with orbit determination accuracies using the aforementioned theories.

Comparison methodology

The following methodology was used in this study: A full range of initial orbit conditions were considered, i.e. a, e and i were altered, to cover the orbital classes mentioned in Table 1. For each initial condition, a theory is used to predict the virtual objects position into the future. Numerically generated orbits are then fitted against the predicted orbits. The details of this methodology are given below.

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*www.celestrak.com

Reference orbit generation and propagation interval: Orbits with different initial conditions are generated by using GP and Semi-analytical theories. The mean orbital arc of five days is obtained by adding variations (periodic and non-periodic) to initial values according to the propagation theories. Using the obtained approximated osculating ephemeris as measurement data, a numerical orbit was fitted against them. RMS of the residuals from orbit fitting are considered to compare the accuracy of theories.

The analysis was performed using ODEM (Orbit Determination for Extended Manoeuvres) software, which is developed at GSOC/DLR. Orbit determination within ODEM is formulated as a sequential non-linear least-squares, with a standard numerical integration method.[2] Shampine & Gordon variable order and variable step-size integrator is employed for orbit prediction. Numerical orbit is generated using a comprehensive model for perturbing accelerations, which comprises of non-spherical gravitational field of the Earth, luni-solar perturbations, atmospheric drag, solar radiation pressure and solid Earth tides.

Requirements for Catalogue maintenance

Keeping track of the tracked space objects requires a moderately accurate propagation unlike for geodynamic applications which ask for sub-meter level accuracy. Here, the main application is to reacquire the objects and predict the probability of collisions with acceptable reliability. For this reason the accuracy of a few metres to a few hundred metres would be sufficient. Also, as the catalogue might contain a few thousands objects (just in GEO regime), propagating and updating the catalogue will require fast propagators with low computational loads. That is why numerical propagator techniques might not come in handy for maintaing the catalogue with low latency.

In the present NORAD element sets, orbital information are provided in the form of TLEs. These elements can be propagated only using the Simplified General Perturbation-4 (SGP4/SDP4) propagators to make use of the complete information provided[3] . In combination, both the TLE and SGP4/SDP4 provide propagation accuracies in the order of kilometre. In case of GEO, they have accuracies slightly above the kilometre range. For the initial comparison of these theories, a test satellite with NORAD ID #23636U (INTELSAT 4) is considered. Its osculating orbital elements at the epoch are given in Table 2.

Table 2. Osculating keplerian elements for test satellite (INTELSAT 4 # 23636U)

Epoch date	10 December 2012
Epoch time	10:45:00.000
a	42165.85 [km]
e	0.001
i	0.0028 [deg]
Ω	240.92 [deg]
ω	124.34 [deg]
M	349.25 [deg]
Period	$\sim$ 24 hours

In the literature[4] authors have discussed the required level of accuracy of orbital information to estimate reliable probability of collision for LEO satellites. Using this information and extrapolating the values to higher altitudes, along with tracking condition limitations in GEO regime, the maximum position error RMS was set to be not more than 200 metres, with radial and cross-track error RMS $\leq$ 100 metres.

Considering the requirements, we are looking for the propagator which can predict the orbits with position error RMS $\leq$ 200 metres and 10 - 20 fold faster than the numerical propagator.

In the section following, brief description of mentioned theories are presented. The result section provides the insight to their performance.

THEORY AND SELECTION

The solution to the problem of orbit propagation is obtained by solving differential Equation 1,

$$\frac{d\vec{c}}{dt} = f(\vec{c}, t) \tag{1}$$

$$\vec{c} = [a, e, i, \Omega, \omega, M]$$

with $\vec{c}$ being the orbital elements vector.

Here $f(\vec{c}, t)$ is the perturbing force function. Depending on how a theory solves/handles perturbing forces, orbit propagation theories can be divided into three categories.

I. Special Perturbation theories(SP) are those that feature application of a high-precision numerical integrator to one of the several relatively complete and accurate formulation of accelerations action on a satellite.[5] This theory is accurate but computationally resource consuming.

II. General perturbation theories(GP) are analytical solutions for perturbed orbital motion. Development of such theories features explicit manual term-by-term formulation in orbital elements of the effects of disturbing functions.[6] The elements at any prediction time can be found immediately, avoiding costly step-by-step numerical integration unlike in SP. In GP, two body equations are replaced with analytical approximations which are inflexible and simplified. Simplification involves truncations and approximations of force models to certain degree and order. Consequently, they are less accurate than SP.

III. There exists a theory in-between SP and GP, know as Semi-analytical theory. The semi-analytical theory is formulated by replacing the conventional equations of motion with: (1) equations of motions for the mean elements and (2) expression for the short periodic motion. This offers much larger integration steps (on the order of one or two steps per day) compared to SP (on the order of hundreds of steps per orbital revolution).[7] Like in SP, semi-analytical theory offers the flexibility to choose between different force models which are to be included in orbit predictions. Conservative and non-conservative forces are modelled and solved separately in this method of propagation. This method of orbit prediction offers the propagation errors ranging between few metres to few kilometres.

Despite their limitations, GP and semi-analytical are the propagators chosen in Space Situational Awareness (SSA) installations as mentioned before, the number of objects tracked make it difficult to use SP techniques.

Perturbations in GEO

At GEO altitudes, drag is absent and therefore there is no need to consider it when propagating objects at that height. Other major forces that are acting on GEO objects are non-spherical Earth's gravity field, third body gravitational forces, and solar radiation pressure if the surface area to mass ratio of the object is large. Figure 1 shows the magnitude of different perturbational accelerations acting at GEO altitudes.

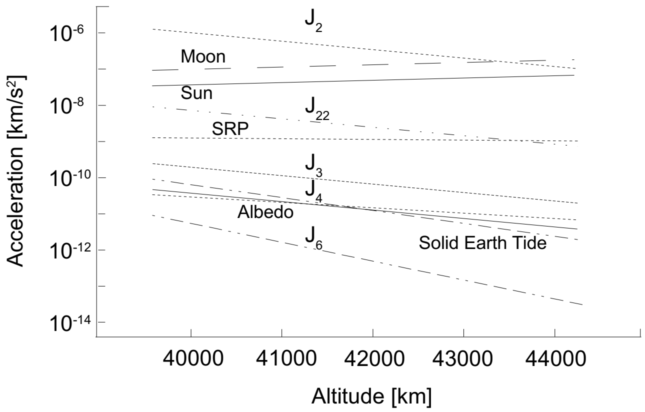

Figure 1. Magnitude of the perturbing accelerations acting at GEO regime

If one chooses to leave out certain perturbations, then an error equivalent to the perturbing acceleration is induced in the propagation and this error grows along with the propagation time. Hence, the inclusion of these forces plays two important roles: 1. provides accurate propagation and 2. allows longer propagation times without deviating from the true track. This was one of the main criteria to chose the theories which will be discussed below.

Selected theories

Theories which include major perturbation accelerations at least down to $10^{-10} km/s^2$ are considered for the study. From the computations from Vallado[8] , considering accelerations up to this magnitude allows the orbit prediction of 5 days for a object in GEO, with deviation of position error less than 10 km. Any propagator which would theoretically perform less than this, will not be worth considering.

Main disturbing accelerations acting upon GEO satellites are: J_2, J_3, luni-solar perturbations, and low degree and order geopotential terms from non-spherical Earth's gravity.

Deep Space Perturbations 4 (SDP4): Simplified general perturbation for Deep Space (SDP4) is an extension of the Simplified General Perturbation-4 (SGP4).[6] SDP4 includes the gravitational effect models of the Sun and Moon. This also extends certain sectoral and tesseral Earth harmonic terms, which are important for GEO orbits.[9]

The perturbational forces that are included in SDP4 are listed in Table 3. After numerical trajectory was fitted against it, residuals in radial(R), along-track(T) and cross-track component(N) are shown in Figure 2. In the plot, behaviour of R and T errors have twice per day (i.e., twice per revolution in GEO case) repetitions, this could be induced from the truncations in short periodic terms of zonal and luni-solar perturbations.[10] N-component shows the behaviour that could be matched with effects caused by tesseral terms.

This theory was included in the comparison, since it is the most widely used propagation technique and the way to make full use of NORAD mean elements. Flavours of SGP were the available theory for the purpose of catalogue maintenance until 1979.[11] The version implemented at GSOC/DLR, which is based on SpaceTrack Report # 3, in Fortran 90 is used in this study.

Table 3. List of perturbations models included in the selected analytical and semi-analytical theories

Perturbations	Theory		
	SDP4	Kamel	DSST
J_2	secular, long periodics, short periodics 0^{th} order in e	secular, long and short periodics	secular, first order long and short periodics
J_3	long periodics	secular, long and short periodics	secular, first order long and short periodics
J_4	secular effects	-	secular, first order long and short periodics
J_2^2	secular effects	secular, long and short periodics	secular, first order zeroth order short periodics in e
Tesseral terms	J_{22},J_{31}, J_{33}	J_{22}, J_{31}, J_{33}	captures up to degree and order 50×50
3^{rd} body	first (P2) term in the Legendre expansion	third order Solar and second order Lunar terms	general luni-solar long and short periodic *
Solar Radiation	-	-	long and short periodics
Others	-	-	solid Earth tides and coupling terms

*The L-S terms in the DSST are general with respect to the parallax factor which is a/R_3 for point mass perturbations

Kamel's theory: Ahmed Aly Kamel's mean element theory is an analytical theory developed for geostationary satellites. This theory is developed by simplifying the equations of motions considerably, due to the fact that geosynchronous satellites are kept near a prescribed orbit in GEOs.[12]

Mainly conservative perturbations are considered in developing the theory; non-spherical Earth's gravity field, and Sun and Moon effects. In extended version of the theory, Kamel adds solar radiation pressure.[13] But here the earlier version without solar radiation pressure is considered for this work. A. A. Kamel uses a specific set of non-singular canonical elements, whics are variations of equinoctial elements to express the perturbation referred to as Kamel Elements. This allows modelling for very low inclinations and eccentricities which is needed for objects in geo stationary orbits. Conversion from Kamel elements to Keplerian and vice-versa can be found in the literature.[12]

Kamel's theory includes the conservative forces which are mentioned in Table 3. An error plot from numerical fit to the theory's orbit is shown in the Figure 3. The selected test-object, is an uncontrolled satellite. Since it is not controlled in a specific orbit, it has small variations in inclinations. Because of this errors are induced in along-track component are observed. The behaviour of N-component is similar to that of the SDP4, which could be caused by truncations in tesseral terms.

The theory derives mean elements averaging which makes use of the satellite's right ascension. This quantity l is given as the sum of Θ – Greenwich hour angle and the nominal sub-satellite longitude λ_{syn}

$$l = \Theta + \lambda_{syn}$$

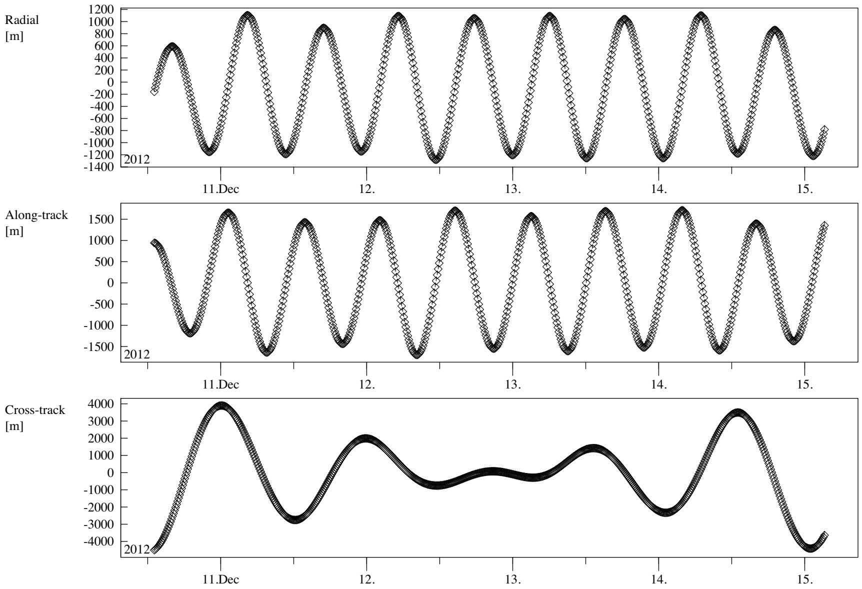

Figure 2. Orbital error of numerically fitted SDP4 orbit in R-T-N components, for the test satellite

That is λ_{syn} is the station right ascension measured from Greenwich along the equator (station longitude). This could be estimated for geostationary orbits quite well. But in case of geo synchronous orbits (with inclination $\neq 0$), it has to be averaged for one orbital period[13] . This induces error into propagator. Apart from this, the theory offers fast initialisation. This could be major advantage for the orbit determination and filtering processes during catalogue maintenance.

Draper Semianalytical Satellite Theory (DSST): DSST is a semi-analytical theory developed by Paul J. Cefola with his colleagues at the Draper Laboratory and Computer Sciences Corporation, Maryland. DSST is a mean element orbit propagator based on the generalised method of averaging. Generalised method of averaging is used to decouple long and short periodic motions. The mathematical development of DSST relies on recursive series to model conservative perturbations and numerical quadratures in modelling non-conservative effects. This allows longer integration time steps for orbit propagation while preserving the accuracy close to SP. Since the integration time step is large in case of DSST, when outputs at small time steps or intermediate times are requested, it makes use of the interpolation strategy. Efficiency of interpolator is important in preserving the efficiency of theory.

The force models included in the theory are given in Table 3. DSST even offers options between geo-potential models. The user has the ability to select between the forces which are to be included, unlike in case of analytical methods. Detailed list and modelling of the perturbations in DSST can be found in Danielson et al,[14] but this document does not provide 'code-to' description of force models.

Unless like SDP4 and Kamel, DSST has many input options to be selected. Optimum input options for GEO orbit propagation was suggested by Fonte.[15] Optimum balance between speed and accuracy option that is set to run test cases with DSST stand-alone propagator are as below:

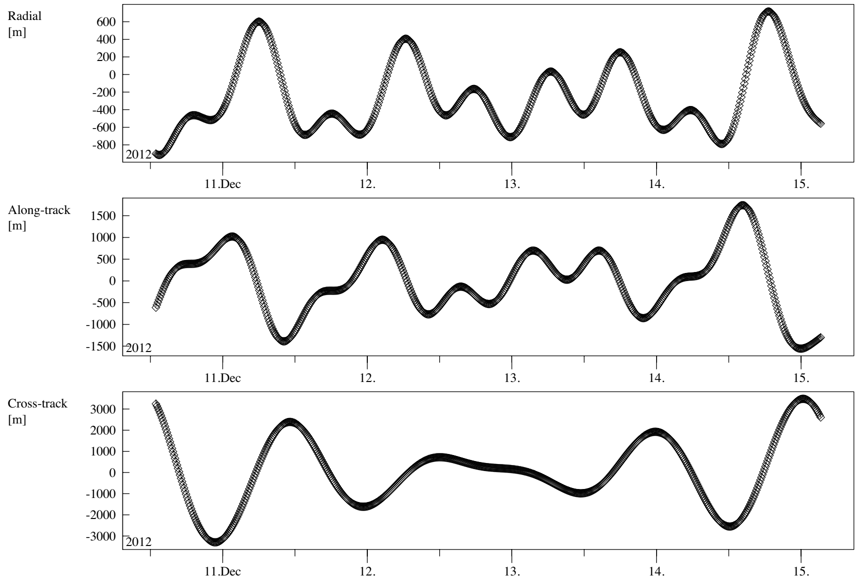

Figure 3. Orbital error of numerically fitted Kamel's orbit in R-T-N components, for the test satellite

- 4×4 geopotential averaged equations
- Averaged and short periodic terms for J_2^2
- Averaged equations for luni-solar point mass effects
- Numerical short periodic terms for luni-solar perturbations

Figure 4 shows the residuals in R-T-N components. Standalone version provided from Cefola, implemented in Fortran 77 is used in this study. This version currently lacks high accuracy setting which is available in GTDS version of DSST.[16] The short insight to GTDS version's accuracy is given in the dedicated section – DSST GTDS version performance.

RESULTS

This section is divided into three parts. The first subsection provides accuracies of the methods when they are fitted with the "true" trajectories generated with different perturbing forces. This gives the insight to the behaviour and modelling limitations of the theories. Tests were conducted for the selected GEO object mentioned in Table2. Second subsection shows the average performance of propagators when a, e and i were varied for the whole sub-class of GEO. The last subsection provides the computational loads of the theories, where these are compared with respect to their runtimes.

Comparison with different perturbing forces

Test cases as below were established to test the best possible least squares numerical fit to the theory. Cases below presents the forces those were included in generation of true orbit.

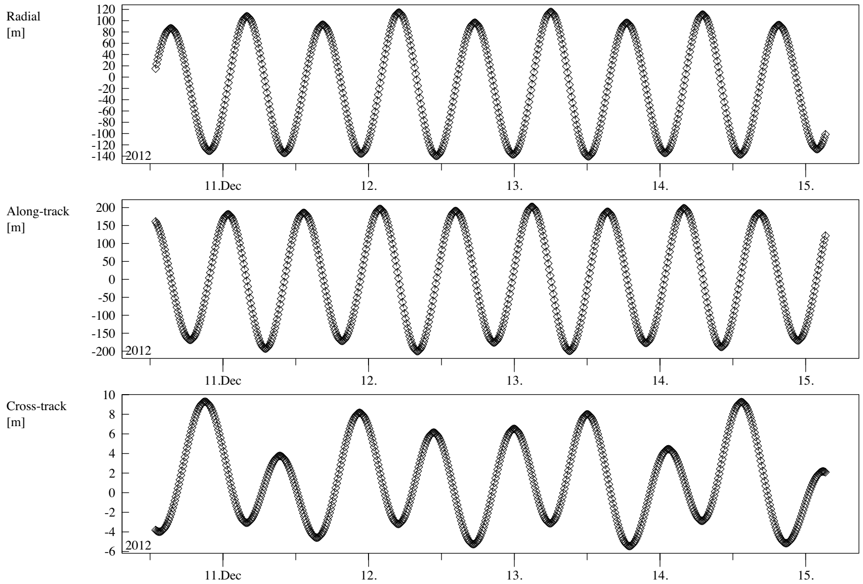

Figure 4. Orbital error of numerically fitted DSST orbit in R-T-N components, for the test satellite

CASE 1 None - Keplerian orbit

CASE 2 J_2 term. i.e, Geopotential model of degree and order of 2×0, without solar and lunar perturbations

CASE 3 J_2 and J_{22}, without solar and lunar perturbations

CASE 4 J_2 , J_{22}, and with solar and lunar perturbations

CASE 5 Including geopotential terms up to 8×8 with solar and lunar perturbations

These tests are conducted to assess the performance of propagator, to the check the modelling of main forces acting on GEO satellites .

For each case, an initial editing level of 1000 meters was set for position vector in orbit determination process. Ephemerides were excluded from fit, for values beyond the editing criteria. For uGEO the orbital elements mentioned in Table 2 are taken. For eGEO, the eccentricity of the test satellite was set to 0.2, and for iGEO inclination of the same satellite was changed to $15°$.

Case-4 is an important case, it provides information close to real world problem. Case-3 is included to see modelling accuracies of luni-solar perturbations, and Case-5 to assess effects of higher order geopotential terms. Tables 4 and 5 gives the performance accuracies for the propagation interval of one and five day prediction lengths.

Modelling of J_2 in SDP4 and Kamel's theory are 0^{th} and 1^{st} order in e respectively. This results in medium to low level accuracy of propagation, which could be noticed in Case-4. Comparing 1-day and 5-day orbital fit errors, it can be observed SDP4 orbits tend to drift away from the true orbit with time. This can be reasoned by the low level truncation of series expansions for secular terms.

Table 4. Theories accuracy comparison for different test cases (1 DAY FIT)

Test cases		SDP4 fit RMS [m]			KAMEL fit RMS [m]			DSST fit RMS [m]		
		uGEO	iGEO	eGEO	uGEO	iGEO	eGEO	uGEO	iGEO	eGEO
CASE 1	R	49.07	34.45	522.71	177.05	1212.61	3534.33	90.66	107.14	310.81
	T	102.28	71.56	385.75	821.92	1549.66	1836.12	206.62	241.13	325.84
	N	192.58	275.19	247.25	265.67	1670.85	223.02	487.79	500.31	237.17
CASE 2	R	49.84	57.95	433.79	176.24	1223.6	3523.72	173.98	336.33	374.88
	T	92.64	117.38	341.65	724.68	1579.35	1550.78	395.94	759.19	782.32
	N	193.88	445.95	250.59	260.77	1742.98	770.02	487.84	321.22	339.14
CASE 3	R	51.02	57.96	429.76	181.39	-	-	170.71	329.43	402. 09
	T	91.56	116.07	339.24	726.30	-	-	393.85	754.23	890.69
	N	193.87	445.47	250.98	260.28	-	-	487.84	320.75	375.71
CASE 4	R	139.20	523.67	632.25	83.22	-	-	82.58	86.78	101.72
	T	92.60	718.90	562.50	129.82	-	-	123.94	130.21	133.27
	N	53.56	79.51	506.40	59.83	-	-	3.36	2.39	7.01
CASE 5	R	139.20	523.58	632.21	85.43	-	-	82.58	87. 89	102.13
	T	92.60	718.94	562.79	129.89	-	-	124.57	130.85	133.28
	N	53.56	79.61	506.62	60.57	-	-	3.93	2.49	7.45

Table 5. Theories accuracy comparison for different test cases (5 DAY FIT)

Test cases		SDP4 fit RMS [m]			KAMEL fit RMS [m]			DSST fit RMS [m]		
		uGEO	iGEO	eGEO	uGEO	iGEO	eGEO	uGEO	iGEO	eGEO
CASE 1	R	314.88	95.03	1225.14	1793.72	-	-	206.66	331.27	330. 93
	T	458.58	496.44	2837.97	1372.19	-	-	437.82	446.66	447.29
	N	1462.32	1714.77	1189.75	2837.82	-	-	471.05	236.65	283.37
CASE 2	R	93.03	94.37	2832.88	617.32	-	-	729.12	330.89	430.32
	T	444.54	497.35	6425.68	1042.01	-	-	1097.56	651.03	832.78
	N	1474.06	2449.66	1255.99	618.64	-	-	524.32	314.41	308.82
CASE 3	R	19.54	17.59	2860.76	439.54	-	-	733.83	353.46	496.98
	T	99.41	117.10	6426.18	864.23	-	-	1210.44	783.75	897.22
	N	1474.02	2447.69	1255.05	430.86	-	-	524.32	312.28	389.43
CASE 4	R	693.50	811.19	963.44	193.61	-	-	84.36	87.75	115.36
	T	1077.48	1131.88	1492.22	209.33	-	-	130.90	137.96	173.68
	N	2737.09	429.63	3314.84	108.97	-	-	4.14	2.64	6.11
CASE 5	R	772.13	812.29	963.26	194.34	-	-	84.36	88.75	115.64
	T	1071.68	1129.74	1474.59	209.79	-	-	136.91	138.97	173.49
	N	2737.09	430.03	3316.56	110.89	-	-	4.41	2.63	6.16

For iGEO and eGEO, SDP4 have poor accuracy in comparison with its own propagation in uGEO case. This is because of the exclusion of higher geopotential terms in its perturbation modelling. 1^{st} order luni-solar perturbations point-mass model in SDP4, seems not to be effective enough to capture the effects of third body accelerations.

In case of Kamel's theory, the prediction accuracy in uGEO is slightly better than that of the SDP4 (comparing Case-4 fit RMS). Inclusion of secular terms of higher gravity terms allows the propagation duration farther than that of SDP4. But Kamel's theory suffer from the limitation of λ_{syn}, because of this iGEO and eGEOs did not converge (for the set editing criteria) even after maximum number of iterations in OD.

The semi-analytical theory, DSST, performed the best in comparison to the considered other two propagators. It should behold that inclusion of long and short periodic variations of third body accelerations has great impact in its cross-track component, and radial component. Also the better modelling of short periodic terms (and integrating over certain step-size) allowed the theory to have longer prediction durations. This can be observed by comparing the fit RMS of Case-4 for 1-day and 5-day orbital lengths.

Accuracy comparison

Orbit predictions for the pseudo-satellites with initial orbital conditions covering the whole range of orbits mentioned in orbital classes Table 1, were done with the selected three theories. Numerical "truth" orbits were then fit to these predicted orbits.

For semi-major axis a grid of 100 km, for eccentricity grid of 0.01, and for inclination grid of 2° were used to cover GEO orbital regime. Mean of the errors for 5-day arc, for the considered three theories are given in Table 6.

Table 6. Theories average accuracy comparison in different subclasses along with the average number of iterations taken to fit numerical orbit

Theory	Class	Fit RMS [m]		
		R	T	N
SDP4	uGEO	713.54	1159.59	1337.53
	iGEO	980.23	1246.67	2194.42
	eGEO	1134.73	1974.89	3208.32
Kamel	uGEO	651.76	929.87	973.62
	iGEO	-	-	-
	eGEO	-	-	-
DSST	uGEO	139.49	213.70	8.66
	iGEO	272.56	295.28	8.49
	eGEO	371.51	254.13	9.86

From the previous result section, as one would expect Kamel theory did not converge for $\sim 90\%$ of iGEO and eGEO objects. SDP4 had 100% acceptance and this study produced results which corroborate the findings from previous tests conducted on SDP4 from different authors.[17] [18] [19]

It was found out that, Kamel's theory fails to converge during OD, beyond the inclination of $\pm$ 5°. This limits the application of the theory to uGEO objects.

The weak time dependent terms are included in the formulation of third bodies in DSST. This assumes that third bodies do not move over the course of time of averaging interval, which is one day in case of GEOs. This assumption can be well suited for LEO or MEO objects, and for GEOs they contribute up to 200 m for 5-day orbital fit, the detailed reasoning and behaviour of DSST w.r.t the argument can be seen in literature.[15] Apart from that, the mean positional error from DSST prediction in all GEO subclasses had 100% acceptance with approximately 30% of SDP4s error.

Runtime comparison

In order to ensure the fare comparison of runtimes, all theories used were implemented in FORTRAN 77/90 which makes use of Intel Fortran compiler. ODEM – orbit determination software also makes use of same programming platform. Hardware with Intel i7 multi-core processor, running with open-suse (64-bit) linux operating system was used.

Table 7 summarises the averages of the CPU times for each ephemeris theory, for orbit prediction of 5-day arc for requested output intervals. Table also provides the percentage comparison with numerical method.

For analytical theories these CPU times are much lower if one point at the end of the prediction interval is of interest. But as of now for fair comparison all theories were requested to provide the outputs at 5, 10 and 30 minutes.

Intuitively, as one would expect, analytical methods have advantage of the speed over semi-analytical, DSST. But, the average run times for the prediction intervals for GEO class show that DSST took about twice the time of the GP theories. In the comparison above, numerical propagator included the perturbation models mentioned in test Case-5.

Table 7. Theories' CPU time consumed for different output intervals for the duration of five days

Theory	Output interval [min]	CPU time [s]	% of Numerical method
SDP4	5	1.767	6.9
	10	1.361	6.8
	30	0.806	4.5
Kamel	5	1.634	6.4
	10	1.487	7.4
	30	1.128	6.3
DSST	5	2.431	10.4
	10	2.109	11.5
	30	1.792	11.7

DSST GTDS VERSION PERFORMANCE

This additional section provides performance of GTDS version of DSST. For the comparison study in previous sections, stand-alone version was used and this version consists of unknown anomalies. To provide the actual performance of DSST, a separate analysis was carried out by Paul J. Cefoal using DSST GTDS version.

To quantify the performance of the DSST GTDS version, a 30 day orbital arc was fit against numerical trajectory and for the test satellite mentioned in Introduction section. The test cases those were established to analyse specially short-periodic models are as below:

DSST CASE 1 None - Keplerian orbit

DSST CASE 2 $J_2 + J_2^2$, without solar and lunar perturbations

DSST CASE 3 $J_2 + J_2^2$, and with solar and lunar perturbation terms via numerical quadrature model, without week time dependent terms

DSST CASE 4 $J_2 + J_2^2$, and with solar and lunar perturbation terms via numerical quadrature model, with two weak time dependent terms; and without editing limits

DSST CASE 5 $J_2 + J_2^2$, and with solar and lunar perturbation terms via numerical quadrature model, with two weak time dependent terms; and with 6-sigma editing of observations.

Table 8. Position RMS of GTDS version

	DSST CASE 1	DSST CASE 2	DSST CASE 3	DSST CASE 4	DSST CASE 5
GTDS	1455.7 m	944.0 m	102.8 m	0.391 m	1.986 m
% accepted data	100	100	100	50	100

Table 8 gives the position RMS for considered test cases. From these results following can be inferred:

1. The several configurations of the DSST GTDS are complete in the mean element motion.
2. When weak time dependent terms are included, the numerical quadrature approach demonstrates much smaller residuals.
3. With smaller residuals it is helpful to use a editing criteria to retain all the observations.

Comparing the performance of GTDS and stand-alone version, it can be said that short periodic model options presently available in stand-alone are to be reviewed.

CONCLUSION

This paper employs an innovative approach to compare a satellite trajectory produced by a selected analytical or semi-analytical satellite theory with a numerical orbit based on realistic force models. In this approach, the selected theory is used to generate a reference trajectory. Such reference trajectories are generated for the NORAD SDP4 and Kamel analytical theories and for the DSST semi-analytical theory. The numerical orbit is generated using the Orbit Determination for Extended Manoeuvres (ODEM) program developed at the GSOC / DLR. The ODEM program uses a least squares fitting process to find a numerical trajectory which best approximate the reference orbit due to the specific analytical or semi-analytical theory. Plots of the Radial, Along-track and Cross-track differences between the reference orbits and the best-fit numerical orbits are given.

The DSST standalone orbit propagator package[16] operating in a Linux environment at the GSOC / DLR is used to generate the DSST reference trajectories. This project provided the opportunity to test the short-periodic models in the DSST Standalone much more extensively than in previous testing efforts. Specifically, bugs were uncovered and resolved in the Newcomb operator database employed by the Tesseral Linear combination short-periodic model and in the luni-solar short-periodic model implementations. Also, the small eccentricity J_2^2 short-periodic was implemented in the

DSST standalone and the development of a J_2 secular/m-daily coupling model was initiated. These updates to the DSST Standalone were tested versus the implementation of the DSST in the comprehensive GTDS orbit determination system. The GTDS DSST implementation has been extensively tested using high accuracy, independently generated GNSS orbits as observation data and using satellite laser ranging as observation data.

Two sets of comparison are presented in this paper. Accuracy comparison shows the moderate accuracy levels of semi-analytical; DSST propagator, to analytical; SDP4 and Kamel's theories. Even though analytical propagators are designed to include certain tesseral and sectoral harmonics of earth gravitational field, they fail to capture significant higher order harmonics, which contributes for the inaccuracy in orbit prediction. The semi-analytical theory which has the capability to include the same force models as that of the SP, performed much better in comparison with analytical theories. DSST performed close to the required accuracy level which was set in requirements for catalogue maintenance, with position error of $\sim 250m$. Also with very good accuracy in N-component was demonstrated; slightly more than the required limit of error in R-component was demonstrated.

The runtime comparison, compared the speed of the three considered propagators. The required computational speed was set to 10% of SP and results showed that analytical methods are faster. At requested output intervals, analytical methods have $\sim 6\%$ of the computational load of numerical method and the DSST had $\sim 10\%$ of numerical methods propagation time, which fulfils the set requirement of space object cataloguing.

Our goal is to suggest an orbit propagator with a balance between speed and accuracy. Among compared theories DSST is suggested to be better option than the current SDP4 theory. With the inclusion of the missing perturbation terms and fixing the luni-solar models in stand-alone version, it could be offered as a suitable propagator for SSA application in GEO regime.

FUTURE WORK

Further research should investigate the performance of propagators, in order to suggest most suited theory for catalogue maintenance of all Earth orbiting space objects. The following tasks are required:

- Inclusion of an analytical weak time dependent (WTD) model in the DSST Standalone.
- Refine and test the pre-stored options for high accuracy (especially the GEO flight regime) in the DSST Standalone program.
- Study of orbit propagation for high surface area to mass ratio GEO objects.
- Evaluation of the partial derivative capabilities provided with the selected analytical and semi-analytical theories. Partial derivatives are required to use the theory to process observation data.
- Review of the assumed SSA requirement to verify that they will support processing of observations of a GEO satellite cluster (such as the Astra cluster[20]) including the correct association of observation with space objects and the detection of electric propulsion station-keeping manoeuvres for GEO satellites.
- Orbit propagation studies for the LEO and MEO orbital regimes.

We currently estimate the errors caused by the neglecting WTD terms in the lunar-solar short periodic to be on the order of 100 metres (see Table 8). With a WTD model, we expect the DSST Standalone to achieve an accuracy of 3 to 4 metres for GEO orbits.

Refining the pre-stored accuracy options will ease the operation of the DSST Standalone for these cases.

High area to mass ratio debris space objects are postulated to occur in the GEO regime. These objects may experience large solar radiation pressure perturbations.

New operational concepts such as GEO clusters and electric propulsion may bring new SSA challenges.

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NOTATION

a	Semi-Major axis
e	Eccentricity
i	Inclination
Ω	Right ascension of ascending node
ω	Argument of perigee
M	Mean anomaly
Θ	Greenwich hour angle

LEO	Low Earth Orbits
MEO	Medium Earth Orbits
GEO	Geosynchronous Orbits
RMS	Root mean square
SSA	Space Situational Awareness
$SDP4$	Simplified general perturbation theory for Deep Space-4
$DSST$	Draper Semi-analytical Satellite theory
SP	Special Perturbations
GP	General Perturbations
OD	Obit Determination

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