HIGH FIDELITY PROPAGATION OF ASTEROIDS USING DROMO-BASED TECHNIQUES

Hodei Urrutxua and Jesús Peláez

Space Dynamics Group, Technical University of Madrid (UPM)

Abstract

There is a well-distinguished group of asteroids for which the roto-translational coupling is known to have a non-negligible effect in the long-term. The study of such asteroids suggests the use of specialized propagation techniques, where perturbation methods make their best. The techniques from which the special regularization method DROMO is derived, have now been extended to the attitude dynamics, with equally remarkable results in terms of speed and accuracy, thus making the combination of these algorithms specially well-suited to deal with the propagation of bodies with strong attitude coupling.

Key words: Orbit propagation; Attitude propagation; Perturbation techniques; DROMO.

1. INTRODUCTION

The solar radiation perturbation acting on small celestial bodies plays an important role in the long term evolution of their trajectories. Such a perturbation affects both aspect of the motion: the orbital and the attitude dynamics of the small body. There is a well-distinguished group of asteroids for which the roto-translational coupling is known to have a non-negligible effect in the long-term evolution of their dynamical state [1]. Such are some of the implications of considering the Yarkovsky and Yorp effects upon the dynamics of certain asteroids. Known asteroids with these features are, for instance Eros, Golevka or Toutatis, to cite a few.

Hence, the study —in the long term— of these type of asteroids could benefit from the joint integration of the orbital and rotational problems. This usually translates into a great loss of performance, since the attitude dynamics equations introduce shorter time scales that force much smaller stepsizes than in the case of just a trajectory propagation. Thus the problem becomes stiff, and one of the many ways to deal with this difficulty is the use of specialized propagation techniques, where perturbation methods make their best.

The Space Dynamics Group[1] (SDG) of the Technical University of Madrid (UPM) developed a special perturbation method, named DROMO, which determines the trajectory of the center of mass of celestial bodies by numerically integrating the equations of motion [2]. DROMO, which combines regularization, linearization and perturbations techniques, is not only competitive but even clearly advantageous in many scenarios, both in speed and accuracy, when it is compared to other traditional propagation methods [3, 4, 5].

Seeking for an equally efficient way of handling the coupled attitude propagation, the techniques from which the special regularization method DROMO is derived, have now been extended to the attitude dynamics problem, with similarly remarkable results in terms of speed and precision [6]. The idea consists on taking advantage of the knowledge of the analytical solution for the unperturbed problem (Euler-Poinsot case) in order to solve the perturbed problem with higher accuracy, using for that a formulation based on perturbation techniques and Euler-Rodrigues parameters.

Thus, the goal is to gather these two algorithms (DROMO and the DROMO-based algorithm for the attitude) into a single propagation code, named AstroDROMO, in order to obtain a global propagation tool with the ability to include, when necessary, both aspects of the asteroid dynamics: the orbital motion of the center of mass of the asteroid, and its attitude dynamics. This tool is, as a consequence, specially well-suited to deal with the propagation of bodies with strong attitude coupling. In fact, even though this tool is being tailored to face the asteroid dynamics problem it can be used in other situation in which the orbital–attitude dynamics coupling is important.

[1] SDG-UPM Website: `sdg.aero.upm.es`

2. DROMO FOR ORBITAL DYNAMICS

The derivation of the DROMO algorithm for the trajectory propagation is fully explained in reference 2 and some recent studies of its performance have been carried out in references 3, 4, 5. However, for sake of completeness, a brief description of the method still seems mandatory in order to expose to the reader its nature, relevant features and performance.

The DROMO formulation is based in a new special perturbation method, where a set of non-classical elements (q_1, q_2, q_3, ε_1^0, ε_2^0, ε_3^0, η^0) is introduced. The last four elements are components of a quaternion related to the attitude of the orbital frame, whereas the first three elements are related to the orbital radius, its time derivative and the angular velocity of the orbital frame.

DROMO provides the time evolution of these elements when the perturbation forces are known, by means of the following set of differential equation:

$$\frac{dq_1}{d\sigma} = +\frac{\sin\sigma}{s^2\, q_3}\, f_{px} + \cos\sigma\, \frac{s+q_3}{s^3\, q_3}\, f_{pz} \qquad \frac{d\varepsilon_1^0}{d\sigma} = -\frac{\lambda(\sigma)}{2}\left(\sin(\sigma-\sigma_0)\,\varepsilon_2^0 + \cos(\sigma-\sigma_0)\,\eta^0\right)$$

$$\frac{dq_2}{d\sigma} = -\frac{\cos\sigma}{s^2\, q_3}\, f_{px} + \sin\sigma\, \frac{s+q_3}{s^3\, q_3}\, f_{pz} \qquad \frac{d\varepsilon_2^0}{d\sigma} = +\frac{\lambda(\sigma)}{2}\left(\sin(\sigma-\sigma_0)\,\varepsilon_1^0 - \cos(\sigma-\sigma_0)\,\varepsilon_3^0\right)$$

$$\frac{dq_3}{d\sigma} = -\frac{1}{s^3}\, f_{pz} \qquad \frac{d\varepsilon_3^0}{d\sigma} = +\frac{\lambda(\sigma)}{2}\left(\cos(\sigma-\sigma_0)\,\varepsilon_2^0 - \sin(\sigma-\sigma_0)\,\eta^0\right)$$

$$\frac{d\tau}{d\sigma} = \frac{1}{s^3\, q_3} \qquad \frac{d\eta^0}{d\sigma} = +\frac{\lambda(\sigma)}{2}\left(\cos(\sigma-\sigma_0)\,\varepsilon_1^0 + \sin(\sigma-\sigma_0)\,\varepsilon_3^0\right)$$

that should be integrated simultaneously with the following algebraic relations, that are needed not only to calculate the right hand sides of the equations above, but also to recover the cartesian position and velocity

$$\lambda(\sigma) = \frac{1}{s^3\, q_3}\, f_{py} \qquad s = q_3 + q_1\, \cos\sigma + q_2\, \sin\sigma$$

$$z = \frac{1}{r} = s\, q_3 \qquad \frac{dr}{d\tau} = q_1\, \sin\sigma - q_2\, \cos\sigma$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \\ \varepsilon_2 \\ \eta \end{Bmatrix} = \begin{bmatrix} \cos(\frac{\sigma-\sigma 0}{2}) & \sin(\frac{\sigma-\sigma 0}{2}) & 0 & 0 \\ -\sin(\frac{\sigma-\sigma 0}{2}) & \cos(\frac{\sigma-\sigma 0}{2}) & 0 & 0 \\ 0 & 0 & \cos(\frac{\sigma-\sigma 0}{2}) & -\sin(\frac{\sigma-\sigma 0}{2}) \\ 0 & 0 & \sin(\frac{\sigma-\sigma 0}{2}) & \cos(\frac{\sigma-\sigma 0}{2}) \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_3^0 \\ \varepsilon_2^0 \\ \eta^0 \end{Bmatrix}$$

where (f_{px}, f_{py}, f_{pz}) are the non-dimensional components of the perturbing force.

Note that the independent integration variable, σ, is not the time but rather a *fictitious time* instead. Hence, the non-dimensional time, τ, is a dependent variable that must be determined by the method itself through integration, yielding a set of 8 differential equations. For an unperturbed problem the independent variable, σ, corresponds to the true anomaly, though for the general case of a perturbed orbit, the physical meaning of σ becomes unclear.

The most remarkable features of the DROMO method are the following:

- Unique formulation for the three types of orbits: elliptic, parabolic and hyperbolic, hence avoiding singularities in the nearby of parabolic motion.
- It uses orbital elements as generalized coordinates; as a consequence, the truncation error vanishes in the unperturbed problem and is scaled by the perturbation itself in the perturbed one.
- The method doesn't have singularities for small inclination nor small eccentricities.
- The orbital plane attitude is determined by Euler-Rodriguez parameters, which are free of singularities and provides easy auto-correction as well as robustness.

- ◇ Easy programming compared to other regularized methods, since the perturbing forces are projected in the orbital frame, making easy use of models proper of Orbital Dynamics
- ◇ No need to solve Kepler's equation in the elliptic case, nor the equivalent for hyperbolic and parabolic cases, since time is one of the dependent variables determined by the method itself.

In the tests carried out so far, the DROMO algorithm has shown a remarkable performance both in terms of speed and accuracy. Also when compared to other special perturbation methods, DROMO has proved to be not only competitive, but very often superior in performance. As a proof of this statement, Figure 1 shows a comparison of the performance of a DROMO propagation versus a direct integration of the Cowell equations, for a test case taken from reference 7. The graph evidences that, for this scenario, the DROMO propagator is faster and more precise than a propagator based on the Störmer-Cowell integrator.

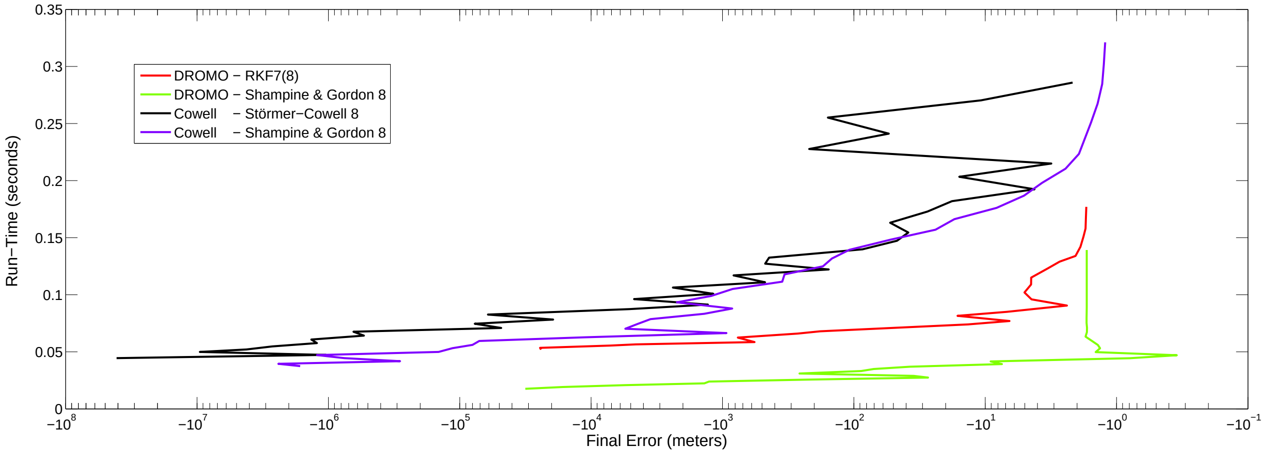

Figure 1. Comparative results showing the"Run-Time" vs "Final Error" relation for different propagators when used in Stiefel & Scheifele's Example 2b, from reference 7.

Even though it is true that the comparison is not always so exaggeratedly favourable to DROMO, the superior performance of DROMO holds for most scenarios studied so far, but further testing is still being carried out in this regards by our research group.

3. DROMO FOR ATTITUDE DYNAMICS

The success of the DROMO algorithm for the trajectory propagation encouraged us to look for an analogue formulation (i.e. based on the same perturbation techniques) for the rotational dynamics of a body. The first promising attempt was presented a few years ago in a conference paper[6] for the attitude propagation of axisymmetric bodies, but its further development was temporarily put aside.

Most recently, this original algorithm has been recovered, some parts have been re-formulated to get rid of singularities and improve its robustness, and the method has also been extended to deal with not only axisymmetric bodies, but also triaxial ones. Therefore, these enhancements we bring in this paper signify a step forward in the development of this method.

3.1. Unperturbed Axisymmetric Problem

The starting point for our formulation is the analytical solution of the unperturbed axisymmetric problem. To begin with, let's consider an inertial reference frame $\mathcal{F}_1 = \{\vec{i}_1, \vec{j}_1, \vec{k}_1\}$ and a body fixed reference frame $\mathcal{F} = \{\vec{i}, \vec{j}, \vec{k}\}$, so the attitude of the body is given by the matrix $\mathcal{S}(t)$

$$[\vec{i}, \vec{j}, \vec{k}] = [\vec{i}_1, \vec{j}_1, \vec{k}_1]\, \mathcal{S}(t)$$

and its angular velocity is given in body frame coordinates as

$$\vec{\omega} = p\,\vec{i} + q\,\vec{j} + r\,\vec{k}$$

Then, the attitude of the body shall be given by the differential equation

$$\frac{\mathrm{d}\mathcal{S}}{\mathrm{d}t} = \mathcal{S}\,\mathcal{W}$$

where $\mathcal{W} = \mathcal{S}^T(t)\,\dot{\mathcal{S}}(t)$ is a skew-symmetric matrix representing the angular velocity $\vec{\omega}$.

Let's also consider an axisymmetric body with two equal inertia moments I_0 and the other equal to $b\,I_0$, where $b \in [0, 2]$ due to the properties of the inertia tensor. The angular momentum is then written as

$$\vec{H} = I_0\,(p\,\vec{i} + q\,\vec{j} + b\,r\,\vec{k})$$

and as for the unperturbed problem the angular momentum remains constant in the inertial frame $\mathcal{F}_1$ (though not in the body frame $\mathcal{F}$), it seems adequate to define a new inertial frame, $\mathcal{F}_0 = \{\vec{i}_0, \vec{j}_0, \vec{k}_0\}$, aligned with the angular momentum such that $\vec{H} = H\,\vec{k}_0$. This introduces a new rotation matrix $\mathcal{Q}$, defined by the relations

$$[\vec{i}, \vec{j}, \vec{k}] = [\vec{i}_0, \vec{j}_0, \vec{k}_0]\,\mathcal{Q}(t), \qquad \mathcal{Q} = \begin{pmatrix} \dfrac{q_0}{\omega_c} & \dfrac{-p_0}{\omega_c} & 0 \\ \dfrac{p_0\,b\,r_0}{\omega_c\,\Omega_c} & \dfrac{q_0\,b\,r_0}{\omega_c\,\Omega_c} & \dfrac{-\omega_c}{\Omega_c} \\ \dfrac{p_0}{\Omega_c} & \dfrac{q_0}{\Omega_c} & \dfrac{b\,r_0}{\Omega_c} \end{pmatrix}$$

with

$$\omega_b = (1-b)\,r_0 \qquad \omega_c = \sqrt{p_0^2 + q_0^2} \qquad \Omega_c = \sqrt{p_0^2 + q_0^2 + (b\,r_0)^2}$$

$$\omega_1 = \frac{1}{2}(\Omega_c + \omega_b) \qquad \omega_2 = \frac{1}{2}(\Omega_c - \omega_b)$$

This matrix $\mathcal{Q}(t)$ relates to $\mathcal{S}(t)$ through their initial values at $t = 0$, according to

$$\mathcal{S}(t) = \mathcal{S}_0\,\mathcal{Q}_0^T\,\mathcal{Q}(t), \qquad \frac{\mathrm{d}\mathcal{Q}}{\mathrm{d}t} = \mathcal{Q}\,\mathcal{W}$$

Note that $\mathcal{Q}_0 = \mathcal{Q}(0)$ is completely defined by the initial angular velocity. Although it becomes undefined if the initial angular velocity is such that $p_0 = q_0 = 0$, which is a trivial case in which the initial angular momentum lies along the body axis, we could set $\mathcal{Q}_0 = \mathcal{S}_0$ in order to avoid the situation.

Now, if we represent the matrix $\mathcal{Q}$ by the Euler-Rodriguez parameters $\mathbf{q} = \{q_1, q_2, q_3, q_4\}$, the analytical solution to the unperturbed axisymmetric problem provides the relations

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} = \Phi(t) \begin{pmatrix} q_{10} \\ q_{20} \\ q_{30} \\ q_{40} \end{pmatrix}, \qquad \Phi(t) = \begin{pmatrix} \cos(\omega_2\,t) & -\sin(\omega_2\,t) & 0 & 0 \\ \sin(\omega_2\,t) & \cos(\omega_2\,t) & 0 & 0 \\ 0 & 0 & \cos(\omega_1\,t) & \sin(\omega_1\,t) \\ 0 & 0 & -\sin(\omega_1\,t) & \cos(\omega_1\,t) \end{pmatrix} \tag{1}$$

$$\begin{aligned} p(t) &= p_0\,\cos(\omega_b\,t) + q_0\,\sin(\omega_b\,t) & p_0 &= 4\,\Omega_c\,q_{10}\,q_{30} \\ q(t) &= -p_0\,\sin(\omega_b\,t) + q_0\,\cos(\omega_b\,t) & q_0 &= 2\,\Omega_c\,(q_{10}\,q_{40} + q_{20}\,q_{30}) \\ r(t) &= r_0 & b\,r_0 &= \Omega_c\,(1 - 2\,(q_{10}^2 + q_{20}^2)) \end{aligned} \tag{2}$$

3.2. Perturbed Axisymmetric Problem

When the axisymmetric problem is perturbed, one can essay solutions of the same type as for the unperturbed problem, like follows

$$\begin{aligned} p(t) &= p_0(t)\,\cos(\omega_b(t)\,t) + q_0(t)\,\sin(\omega_b(t)\,t) \\ q(t) &= -p_0(t)\,\sin(\omega_b(t)\,t) + q_0(t)\,\cos(\omega_b(t)\,t) \\ r(t) &= r_0(t) \end{aligned}$$

where the constants of the unperturbed problem (p_0, q_0, r_0, $\mathcal{Q}_0$, $\mathcal{S}_0$ and derived quantities) have now become variables that evolve with time. So, the objective of this perturbation technique is to integrate in time not the actual attitude and angular velocity variables, but to derive instead the differential equation for these formerly constant variables identifying the initial state of an equivalent unperturbed axisymmetric problem. The advantage of these new variables is that they evolve in a slower time scale, and even become constant when no external torque is applied. The detailed derivation of these expressions is beyond the scope of the current paper, so we shall directly proceed to expose the final set of differential equations for the DROMO method:

$$I_0 \begin{pmatrix} \dot{\Omega}_c \\ \Omega_c\,\dot{q_{10}} \\ \Omega_c\,\dot{q_{20}} \\ \Omega_c\,\dot{q_{30}} \\ \Omega_c\,\dot{q_{40}} \end{pmatrix} = \begin{pmatrix} k_{01} & k_{02} & k_{03} \\ k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \\ k_{41} & k_{42} & k_{43} \end{pmatrix} \begin{pmatrix} \cos(\omega_b\,t) & -\sin(\omega_b\,t) & 0 \\ \sin(\omega_b\,t) & \cos(\omega_b\,t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} L \\ M \\ N \end{pmatrix} \tag{3}$$

$$\frac{\mathrm{d}\mathbf{s}_0}{\mathrm{d}t} = \frac{1}{2}\,\mathbf{s}_0\,(2\,\mathbf{\Omega_p}) \tag{4}$$

with

$$\mathbf{\Omega_p} = -\left\{\tilde{\mathbf{q}}_0(t)\,\left[\Phi\,\dot{\mathbf{q}}_0(t) + \left(\dot{\omega_1}\,\frac{\partial\Phi}{\partial\omega_1} + \dot{\omega_2}\,\frac{\partial\Phi}{\partial\omega_2}\right)\,\mathbf{q}_0(t)\right]\,\tilde{\mathbf{q}}(t)\,\mathbf{q}_0(t) + \dot{\tilde{\mathbf{q}}}_0(t)\,\mathbf{q}_0(t)\right\}$$

$$k_{01} = 2\,(q_{10}\,q_{30} - q_{20}\,q_{40}) \qquad k_{02} = 2\,(q_{20}\,q_{30} + q_{10}\,q_{40}) \qquad k_{03} = 1 - 2\,(q_{10}^2 + q_{20}^2)$$

$$k_{11} = \frac{q_{30}}{4}\cdot\frac{1 - 2\,q_{10}^2 - 6\,q_{40}^2 + 8\,q_{40}^4 + 8\,q_{30}^2\,q_{40}^2}{q_{30}^2 + q_{40}^2} \qquad k_{21} = -\frac{q_{40}}{4}\cdot\frac{1 - 2\,q_{20}^2 - 6\,q_{30}^2 + 8\,q_{30}^4 + 8\,q_{30}^2\,q_{40}^2}{q_{30}^2 + q_{40}^2}$$

$$k_{31} = \frac{q_{10}}{4}\cdot\frac{1 - 2\,q_{30}^2 - 6\,q_{20}^2 + 8\,q_{20}^4 + 8\,q_{10}^2\,q_{20}^2}{q_{10}^2 + q_{20}^2} \qquad k_{41} = -\frac{q_{20}}{4}\cdot\frac{1 - 2\,q_{40}^2 - 6\,q_{10}^2 + 8\,q_{10}^4 + 8\,q_{10}^2\,q_{20}^2}{q_{10}^2 + q_{20}^2}$$

$$k_{12} = \frac{q_{40}}{2}\cdot\frac{q_{20}^2 - q_{40}^2 + 2\,q_{30}^2 + 2\,q_{40}^4 - 2\,q_{30}^4}{q_{30}^2 + q_{40}^2} \qquad k_{22} = \frac{q_{30}}{2}\cdot\frac{q_{10}^2 - q_{30}^2 + 2\,q_{40}^2 + 2\,q_{30}^4 - 2\,q_{40}^4}{q_{30}^2 + q_{40}^2}$$

$$k_{32} = \frac{q_{20}}{2}\cdot\frac{q_{40}^2 - q_{20}^2 + 2\,q_{10}^2 + 2\,q_{20}^4 - 2\,q_{10}^4}{q_{10}^2 + q_{20}^2} \qquad k_{42} = \frac{q_{10}}{2}\cdot\frac{q_{30}^2 - q_{10}^2 + 2\,q_{20}^2 + 2\,q_{10}^4 - 2\,q_{20}^4}{q_{10}^2 + q_{20}^2}$$

$$k_{13} = -\frac{q_{20}}{2}\cdot\frac{1-b}{b}\,\Omega_c\,t - (q_{30}^2 + q_{40}^2)\,q_{10} \qquad k_{23} = \frac{q_{10}}{2}\cdot\frac{1-b}{b}\,\Omega_c\,t - (q_{30}^2 + q_{40}^2)\,q_{20}$$

$$k_{33} = -\frac{q_{40}}{2}\cdot\frac{1-b}{b}\,\Omega_c\,t + (q_{10}^2 + q_{20}^2)\,q_{30} \qquad k_{43} = \frac{q_{30}}{2}\cdot\frac{1-b}{b}\,\Omega_c\,t + (q_{10}^2 + q_{20}^2)\,q_{40}$$

where $\{L, M, N\}$ are the body frame coordinates of the external torque, and quaternion algebra notation was used to express the relations as compactly as possible. The attitude of the body is then given by

$$\mathbf{s}(t) = \mathbf{s}_0(t)\,\tilde{\mathbf{q}}_0(t)\,\mathbf{q}(t)$$

whith $\mathbf{q}(t)$ obtained from Eq (1), and the angular velocity is provided by Eq (2).

Note that Eqs (3) and (4) form a set of 9 differential equations, but the order of the system can be reduced since not all of them are independent. In fact, Eq (4) propagate the four components of the quaternion $\mathbf{s}_0$, so one of these equations is a linear combination of the other three. However, for robustness and auto-correction issues, it is advisable to integrate all the four componets. On the other side, the five components of the vectorial equation (3) are the equivalent of the angular velocity, so two of these variables are a linear combination of the other three. Here, it turns out necessary to eliminate these redundancies in order to avoid singularities in the components of matrix K, but the variables to be excluded depend on the problem:

- ◇ If the angular momentum vector gets too close to the axisymmetry axis, then q_{10} and q_{20} tend to zero and elements k_{31}, k_{41}, k_{32}, k_{42} tend to infinity, making the K matrix ill-conditioned of even singular. In this case, not computing the time derivatives of q_{30} and q_{40} eliminates the problem, since these quaternions can be obtained from the relations

$$q_{30} = \pm q_{20} \sqrt{\frac{1 - q_{10}^2 - q_{20}^2}{q_{10}^2 + q_{20}^2}}, \qquad q_{40} = \mp q_{10} \sqrt{\frac{1 - q_{10}^2 - q_{20}^2}{q_{10}^2 + q_{20}^2}}$$

  where the sign is chosen consistently with the initial conditions.

- ◇ If the angular momentum vector gets nearly perpendicular to the axisymmetry axis, then q_{30} and q_{40} could both get close to zero and hence elements k_{11}, k_{21}, k_{12}, k_{22} would tend to infinity. In this case, not computing the time derivatives of q_{10} and q_{20} eliminates the problem, since these quaternions can be obtained from the relations

$$q_{10} = \pm q_{40} \sqrt{\frac{1 - q_{30}^2 - q_{40}^2}{q_{30}^2 + q_{40}^2}}, \qquad q_{20} = \mp q_{30} \sqrt{\frac{1 - q_{30}^2 - q_{40}^2}{q_{30}^2 + q_{40}^2}}$$

  where the sign is chosen consistently with the initial conditions.

Therefore, the order of the DROMO equations finally reduces to a set of 7 ordinary differential equations.

3.3. Perturbed Triaxial Problem

Mathematically, every triaxial problem can be converted into an equivalent perturbed axisymmetric problem.

Let's consider a triaxial body with inertia moments $\{I_x, I_y, I_z\}$. If we assume, without loss of generality, that the closermost inertia moments are I_x and I_y, then one can apply the following change of variables

$$I_0 = \frac{I_x + I_y}{2}, \qquad a = \frac{I_x - I_y}{2\, I_0}, \qquad b = \frac{I_z}{I_0}$$

where I_0 is the radial inertia moment of an equivalent axisymmetric problem, a is a parameter measuring the non-axisymmetry, and b is a parameter measuring the oblateness or prolateness of the mass geometry. The parameter a seems to be bounded between ± 1, where $a = 0$ corresponds to the axisymmetric case; however, the hypothesis that I_x and I_y should be the closermost moments imposes a constraint to the values of a, relating it to b. Additionally, it is a property of the inertia tensor that the sum of two inertia moments must be equal or greater than the third moment. Hence, a and b turn out to be bounded as

$$b \in [0, 2], \qquad |a| \leqslant \frac{|1 - b|}{3} \leqslant \frac{1}{3}$$

Thus, a perturbed triaxial problem where the torque $\vec{M} = \{L, M, N\}$ is applied, can be converted into an equivalent axisymmetric problem with the new torque $\vec{M}' = \{L', M', N'\}$. This torque can be proven to satisfy the relations

$$L' = \frac{1}{1+a}\big(L - a\,(2-b)\,I_0\,q\,r\big), \qquad M' = \frac{1}{1-a}\big(M - a\,(2-b)\,I_0\,p\,r\big), \qquad N' = N + 2\,a\,I_0\,p\,q$$

where $\{p, q, r\}$ are the angular velocity components in body frame coordinates.

This transformation allows us to use the DROMO method, originally designed for axisymmetric problems, also for triaxial bodies, thus extending its applicability. The price to pay is to include a fictitious perturbation that takes into account the triaxiality or non-axisymmetry of the problem, measured by the parameter a.

This affects the performance of the DROMO method proportionally to the magnitude of this non-axisymmetry perturbation. Hence, nearly axisymmetric bodies will be very well suited for DROMO method, whereas the propagation of highly triaxial bodies might even become inefficient in worst cases. However, the extent of this loss performances is still under evaluation.

As the unperturbed triaxial problem, also known as Euler-Poinsot problem, has an analytic solution, this is potentially a good test scenario to evaluate and quantify the performance of DROMO for non-axisymmetric bodies. Figure 2 shows the performance of DROMO with respect to the analytic solution, where the vertical axis stands for the number of function

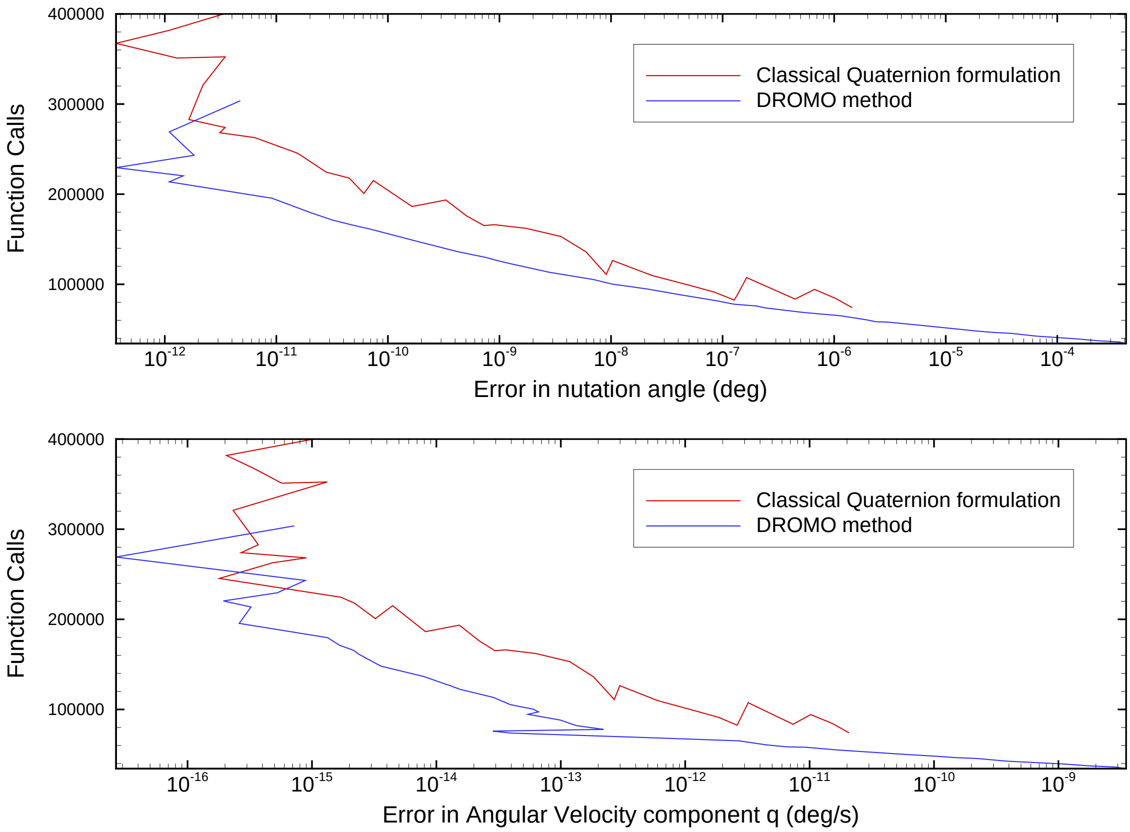

Figure 2. Comparative results showing number of function calls versus the final error with respect to the analytical solution of the Euler-Poinsot problem, for 1) the direct integration of Euler equations and quaternions, and 2) the DROMO method. This scenario corresponds to a mass geometry defined by $I_0 = 4.5 \cdot 10^3$*,* $a = 0.02$*,* $b = 0.666$*, initial euler angles* $\{30, 40, 50\}$ *degree and body frame angular velocity* $\{p, q, r\} = \{1, 1, 60\} \cdot 10^{-5}$ *deg/s, integrated for approximately for 1000 periods.*

calls and the horizontal axis represents, for the blue line, the error between the DROMO solution and the analytical solution. In the plot we have also included a red line, which shows, for comparison, the performance of the direct numerical integration of the Euler equations along with quaternions to propagate the attitude.

Preliminary tests show that the loss of performance of DROMO might get significant with respect to a purely axisymmetric problem, where speed-ups of an order of magnitude are common. Nevertheless, for slightly triaxial problems DROMO still shows a remarkable performance, but further study is yet necessary to understand and quantify the goodness and the limitations of the method.

4. CONCLUSIONS

So far, we have introduced the DROMO formulations for the orbital and attitude dynamics, and we have presented proofs of their performance, which show that these techniques might offer an outstanding combination of speed and accuracy, compared to traditional propagation techniques.

The improvement and testing of these methods is currently an ongoing work in the SDG-UPM research group, so the results discussed in this paper are just preliminary. In the following months, further refinements of the algorithms and more detailed comparative results are expected.

However, these first results look promising, and therefore we believe that the combination of these DROMO methods could be a great asset for the fast and accurate propagation of roto-translationally coupled bodies, becoming a powerful tool for the numerical propagation of their dynamics.

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