Use of Rotating Space Tethers in the Exploration of Celestial Bodies

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This article gives an insight into the potential applications of rotating space tethers for the exploration of planetary satellites, pursuing a semi-analytic model that permits to study the influence of a tether in the design of orbits of interest for science missions. The averaging method allows to remove the fast time scales related to the tether's rotation and the orbital motion, yielding to a model that successfully describes the very long term evolution of a tethered system for over months of mission. This model eases an initial analysis of the long term evolution of the tether's attitude, unveiling a precession phenomenon of its rotation plane. Additionally, the model is applied to the search for frozen orbits, revealing promising orbit stabilization features that allow for the modification of frozen orbits by purely mechanical means, leading to lower eccentricity orbits for given altitude. Hence, it turns out that the length of the tether becomes an additional design parameter that shapes frozen orbits to fulfil tighter operational constraints.

I. Motivation

The exploration of planetary satellites by robotic spacecraft is currently of strong scientific interest. One of the challenges of planning such a mission is the design of the science orbit,[1] which is the orbit where the acquisition of scientific data takes place. Science orbits for missions to planetary satellites have, in general, low altitudes and near-polar inclinations so that the entire surface can be mapped, and the science requirements of the mission can be accomplished.[2] However, designing such an orbit can be difficult because the dynamical environment of many planetary satellites is highly perturbed with respect to an integrable two-body system due to their proximity to their central planet.[3] One such example is the Moon, on which we focus on the current study.

High-inclination orbits around the Moon are known to be unstable,[4,5] and thus emerges the problem of maximizing the orbital lifetime.[6] At this concern, the so called *frozen* orbits offer an interesting starting point for the design of science orbits,[1,7,8] for they have the peculiarity that the orbital elements (with the exception of the mean anomaly and longitude of the ascending node) remain constant. Therefore, the search for high inclination and low eccentricity frozen orbits is a very attractive research field.

Hence, the aim of this work is the obtention of a reliable model that adequately describes the long and very long term evolution of a tethered system around a planetary satellite, permiting an analysis of its dynamics over long periods of time, as well as the application of this model to the search for frozen orbits. The whole content of these article is based upon the results achieved within the MSc Project.[9]

I what follows, we first introduce the concept of a tethered system and obtain the equations governing its long term evolution. Then, we draw some conclusions attained in relation to a precession phenomenon occurring to the tether's rotation plane, and finally we proceed to analyze how frozen orbits are modified by the presence of a rotating tether.

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II. The Tethered System

A space tether[10, 11] is basically a rope or a cable that is floating in space. Obviously, a tether on its own is quite useless unless we attach it somewhere. The real interest of tethers resides on the possibility of tying or binding two different bodies in space, creating a leash or constraint between both whenever the tether is tight. So, we shall call a tethered satellite, or by extension simply a tether, to a system compounded by one, two or more satellites tied to each other by a cable or a string that imposes a restraint to the dynamics of the system. Note that space tethers are usually many kilometers long, for their length is a key design parameter that offers greater practical advantages when it is longer.

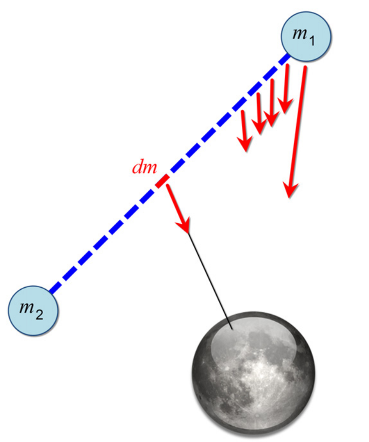


Figure 1: Scheme illustrating the Full Two-Body approach.

The idea of a tether is then conceived as a constraint. Therefore, it is expected that the tether stays tight during its operation, which actually happens in most of applications due to the tension on the cable, that arises whether from the gravitational gradient in librating tethers, or the centrifugal force in rotating tethers. This being a very realistic situation for a rotating tether, a first idea aroses of considering the tether as a rigid element that accomplishes the same constraint upon the end-masses or satellites attached at the tether. In other words, we could consider the tether to be a rigid rod instead of a flexible cable that might get loose. This model is known as the *dumbbell model*, and is the most convenient one for the current study, for its greatest advantage is the resulting simple analytical formulation, that is easily handled by means of classical mechanics.

For an accurate enough analysis of the stability of the resulting orbits, we are challenged to obtain general expressions for the gravitational actions upon a tethered system produced by a non-uniform gravity field. The gravitational potential and torque, with the only restraint that the non-uniform gravity field will be assumed to have revolution symmetry (only zonal harmonics are to be retained), are obtained under the formulation of the *restricted* two-body problem. There are many different approaches one might take to this problem, each having a different degree of complexity and being more or less adequate for a certain aim.

The most general approach is known as the *Full Two-Body Problem*, where both bodies are considered to be extensive. Every mass element $\mathrm{d}m$ of a body is attracted by every $\mathrm{d}m$ of the other body. Consequently, a double volume integration is needed. This is the required approach, since we are interested in considering the attracting celestial body as extensive, and we intend the tether to be also an extensive body affected by gravity forces and torques.

The full two-body approach is usually avoided unless it is really necessary, due to the inherent difficulty of the double volume integral, leading to complicated sets of equations. However, when one of the two bodies is so slim as a tether, this body would very well admit to be treated as a linear body, and one of the volume integrals would turn into a linear integral, greately simplifying the resulting equations.[11]

The first milestone of our work has been the obtention of general expressions for the gravitational actions upon the tether, that includes an arbitrary number of zonal harmonics under the Full Two-Body approach. These expressions are not found in the bibliography, so they had to be developed from scratch, since are essential for the accurate description of the dynamical system. In spite of having general expressions, for the sake of simplicity just the J_2 harmonic is retained in the study exposed in this article.

III. Long Term Evolution of the Tethered System

According to classical mechanics, a tethered system considered as a rigid extensive body has six degrees of freedom; three of them are needed to define its spatial location, while the other three are necessary to describe its attitude or orientation in space. These degrees of freedom are represented by three spatial coordinates (x, y, z) that define the motion of the center of mass of the tether (translational problem), and three angular coordinates (ϕ_1, ϕ_2, ϕ_3) (representing the Tait-Byan rotation angles) that describe its rotational state (rotational problem).

The time variation of these six coordinates is ruled by a system of six second degree ordinary differential equations, although the rotational problem might be very conveniently transformed from a set of three second degree differential equations to four first degree differential equations. The problematic part is that for a tether, the translational and the rotational problems are coupled, so the whole system has to be integrated at once.

It is easy to notice that these equations evolve in several different time scales, so that they become susceptible of performing various averaging processes, by means of which the motion in the *slow* time scales might be simplified by averaging the motion in the *fast* variables of the problem.

The fastest time scale present underneath our dynamical problem is one related the tether's self-rotation, $\Omega_\perp$. The characteristic time of the tether's rotation is the inverse of $\Omega_\perp$. The next larger time scale is the one associated to the tether's orbital motion around the Moon, usually the mean orbital motion $n = \sqrt{\mu_{\leftmoon}/a^3}$, whose characteristic time is the orbital period. So, the relation $\Omega_\perp/n \gg 1$ permits to apply the averaging method to the equations of motion,[11] in order to find the time evolution of the system in the slow time scale $n\,t \sim \mathcal{O}(1)$.

Additionally, we also realize that the variation of some orbital elements, as well as the tether's plane of rotation, is a slow process that takes place almost inadvertently along many revolutions around the primary body, in our case the Moon. This means that the time scale of an orbital revolution is much smaller than the time scale in which some orbital elements vary. This brings the possibility of making a second averaging of the equations, in order to obtain simplified expressions that would provide the long term variation of the orbital elements. The equations governing the attitude of the tether will also find advantages in this second averaging, for the Tait-Bryan angles ϕ_1 and ϕ_2 are also slowly changing variables, that barely vary along a single orbit due to the fact that a fast rotating tether has a stabilized attitude. However, it will be revealed that in long term propagations the attitude is exposed to great variations that take place along weeks and even months.

The consideration of the Earth as a third body perturbation introduces another time scale related to the motion of the Moon around the Earth. As this motion is periodic and faster than the variation of some orbital elements, we may average the Earth perturbation for a third time to provide simpler equations that retain the essence of the very long term evolution of the dynamics.

After arduous manipulations performed in simplifying and averaging the perturbing terms in the equations of motion, it turns out that the equations that rule the long term attitude dynamics of the tether are

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}\tau} = \left(\frac{n}{\Omega_\perp}\right)\frac{3}{4}\,\frac{1}{\cos\phi_2}\,\frac{\mathbb{A}\,\mathbb{E}+\mathbb{B}\,\mathbb{F}}{(1-e^2)^{\frac{3}{2}}} \tag{1}$$

$$\frac{\mathrm{d}\phi_2}{\mathrm{d}\tau} = \left(\frac{n}{\Omega_\perp}\right)\frac{3}{4}\,\frac{\mathbb{C}\,\mathbb{E}+\mathbb{D}\,\mathbb{F}}{(1-e^2)^{\frac{3}{2}}} \tag{2}$$

$$\frac{\mathrm{d}\phi_3}{\mathrm{d}\tau} = \left(\frac{\Omega_\perp}{n}\right) - \left(\frac{n}{\Omega_\perp}\right)\frac{3}{4}\,\tan\phi_2\,\frac{\mathbb{A}\,\mathbb{E}+\mathbb{B}\,\mathbb{F}}{(1-e^2)^{\frac{3}{2}}} \tag{3}$$

$$\frac{\mathrm{d}\Omega_\perp}{\mathrm{d}\tau} = 0 \tag{4}$$

where coefficients $\mathbb{A}$ to $\mathbb{F}$ are intricate functions of the orbital elements and the Tait-Bryan angles, that can be found in the MSc Project.[9]

We can of course check their validity upon numerical simulations (fig. 2a) and see how these averaged equations remove from the angle ϕ_1 the tinny oscillations due to the orbital motion, and leave at sight just the long-term evolution of the angle.

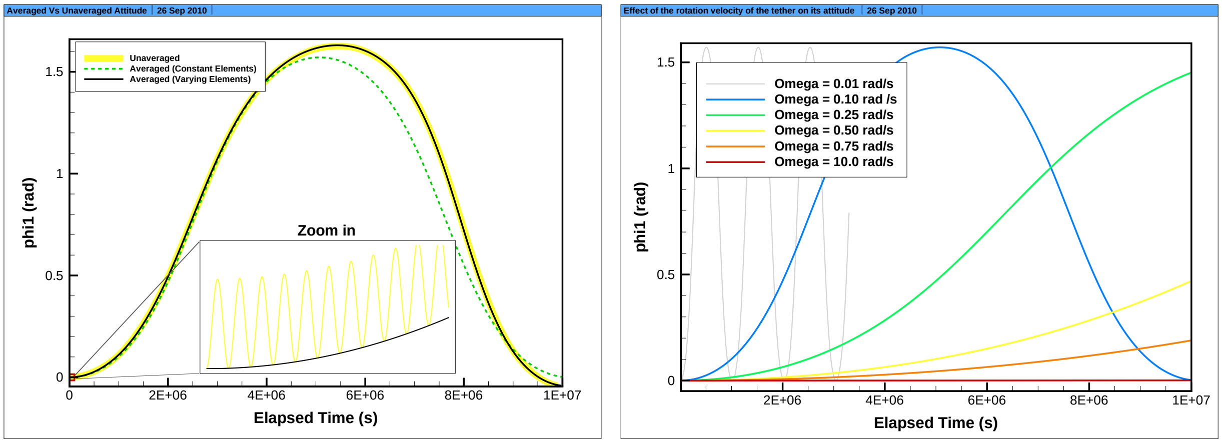
Another interesting conclusion from figure 2a arises from the comparison of the averaged equations with varying and non-varying orbital elements (respectively black and green lines). Their deviation indicates that the time scale in which the orbital elements change and the tether's attitude change might be similar, and if this was the case, the rotational and translational problems would remain coupled for their long term evolution as well.

As we have already discussed, for a fast rotating tether we usually assume

$$\frac{n}{\Omega_\perp} \ll 1$$

which explains that equations (1) to (3) are often reduced to

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}\tau} \simeq 0 \qquad \frac{\mathrm{d}\phi_2}{\mathrm{d}\tau} \simeq 0 \qquad \frac{\mathrm{d}\phi_3}{\mathrm{d}\tau} \simeq \frac{\Omega_\perp}{n}$$



(a) (b)

Figure 2: **Above:** Comparison between the averaged and unaveraged equations of the attitude. Two versions of the averaged equations are shown, one where the orbital elements have been considered costant (green), and the other obtained by integrating the roto-translational problem in which the orbital elements vary (black). The unaveraged solution (yellow) was also obtained with varying elements. **Below:** The effect that the increasing rotational velocity of the tether $\Omega_\perp$ has in the period in which the attitude of the tether (angles ϕ_1 and ϕ_2) changes.

and so ϕ_1 and ϕ_2 are to be considered constant. This approximation allows the decoupling of the rotational and the translational problems by assuming that the tether's plane of rotation does not vary, since it is stabilized by the tethers rotation. This stabilization, and thus the validity of this approximation actually depends on the ratio $n/\Omega_\perp$, i.e. the faster the tether rotates, the more constant the angles ϕ_1 and ϕ_2 can be considered.[12] The figure 2b shows an applied numerical example.

The translational part of the problem should be equally averaged in an orbit to obtain the long-term evolution of the orbit. Therefore, the most convenient is to employ equations of the motion in a *Variation of Orbital Elements* form. We found adequate to use the Lagrange Planetary Equations.[13–15]

The issue is then to determine the averaged perturbing potential, which can be broken down as the sum of two terms, one due to the gravitational potential of a fast rotating tether in a non-uniform gravity field, and the other due to the presence of the Earth as a third body perturbation[a].

$$\langle \mathcal{R} \rangle = \langle \mathcal{R}_g \rangle + \langle \mathcal{R}_\oplus \rangle$$

The averaging process leads to the following expressions for the above perturbing potentials

$$\langle \mathcal{R}_g \rangle = \frac{\mu_☾ \, m \, a_2 \, L_T^2}{a^3 \, (1-e^2)^{\frac{3}{2}}} \left[\frac{1}{2} - \frac{3}{8} (\mathbb{A}^2 + \mathbb{B}^2 + \mathbb{C}^2 + \mathbb{D}^2) \right] + \frac{\mu_☾ \, m \, J_2 \, R^2}{a^3 \, (1-e^2)^{\frac{3}{2}}} \left[\frac{3}{4} \sin^2(i) - \frac{1}{2} \right] \tag{5}$$

$$\langle \mathcal{R}_\oplus \rangle = - \frac{\mu_\oplus \, m}{\rho^3_{\oplus ☾}} \, a^2 \, \frac{3}{4} \left[\langle \mathbb{I}^2 \rangle \, (1 + 4\, e^2) + \langle \mathbb{J}^2 \rangle \, (1 - e^2) - \left(\frac{2}{3} + e^2 \right) \right] \tag{6}$$

where coefficients $\mathbb{A}$ to $\mathbb{J}$ are intricate functions of the orbital elements Ω, ω, i and the Tait-Bryan angles ϕ_1, ϕ_2, as exposed in the MSc Project.[9]

IV. Precession of the Tether's Rotation Plane

For fast rotating tethers it is usually assumed that $n/\Omega_\perp \simeq 0$, leading to the attitude of the tether's rotation plane to be constant. This is fine for propagations lasting a few days, but for long term propagations this might not be an adequate assumption, for perturbations, when acting for long periods, they do accumulate and can have a secular effect on the dynamics. Simulations reveal that considering the tether's

[a]Note that in this preliminary study we omit the third body perturbation of the Sun, and other minor orbital perturbations.[16,17]

rotation plane constant in time could lead to catastrophic results in long term, for it can really evolve to drift up to 90° or more in a few weeks time. This drift could be delayed by making the tether rotate faster, but we could reach technological limits in the tether deployment or mission constraints that refuse using this solution.

In cases where neglecting the torques is not correct and the future attitude of the tether's plane of rotation is required to be precisely known, equations (1 - 4) need to be solved without neglecting their right-hand side terms. This set of equations provide the long term and even the very long term evolution of the tether's rotation plane. To understand how the rotation plane is expected to evolve, we must carefully study these equations.

Equation (4) indicates that the perturbations considered do not alter in average the component of the rotational velocity of the tether that is perperndicular to the rotation plane, that is, $\Omega_\perp$ is constant. Additionally, equations (1) and (2) are independent from ϕ_3, so the equation (3) can be independently solved, and then our equations system can be reduced to two, these being eqs. (1) & (2), governing respectively the long term evolution of ϕ_1 and ϕ_2.

We are intested in analyzing the qualitative behaviour of the tether's plane of rotation, which is defined by the angles ϕ_1 and ϕ_2. However, these equations are complicated functions of orbital elements a, e, i, Ω and ω, making their general solution non-trivial. Therefore, if we wish to obtain any information from them, we should try to particularize them for cases that would allow great simplifications.

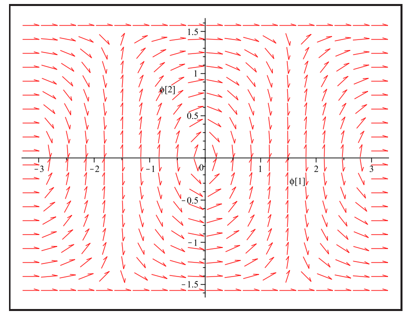
Hence, considering an equatorial orbit ($i = 0$) equations (1) and (2) drastically reduce, and any dependency with the orbital elements is removed. In this situation, a first integral of the motion can be obtained, that solves to provide the relation

$$\cos\phi_1 \, \cos\phi_2 = \cos\Phi \tag{7}$$

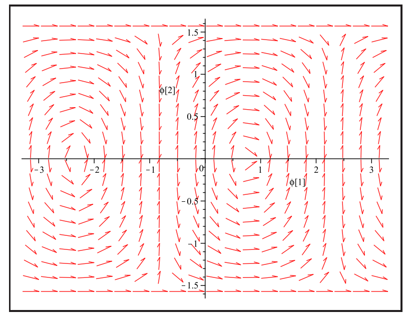
where Φ is a constant that can be determined from the initial values of ϕ_1 and ϕ_2. Φ is easily identifiable as the angle between the tether's angular momentum and the direction perpendicular to the orbital plane (the equator).

This reveals that given a certain initial position of the rotation plane (given a value to Φ), the angles ϕ_1 and ϕ_2 will evolve along the locus of eq. (7).

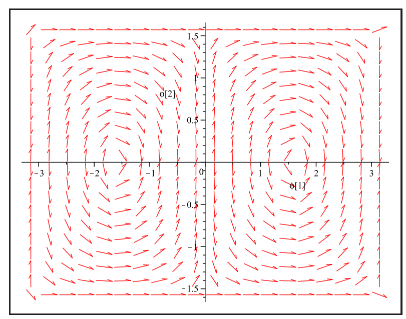
Further information is obtainable from plotting instead the direction field of the resulting differential equation, shown in figure 3a, which unveils that the attitude of the rotation plane is such that it evolves clockwise in a ϕ_1 - ϕ_2 diagram.



(a) $i = 0$



(b) $0 < i < \pi/2$



(c) $i = \pi/2$

Figure 3: Direction field of the differential equations $\frac{\mathrm{d}\phi_2}{\mathrm{d}\phi_1}$ for different values of the orbital inclination.

The most important fact is that the variations of ϕ_1 and ϕ_2 are stable, i.e. given an initial state, the plane of rotation of the tether will evolve in a manner that its angular momentum will preserve the very same angle with respect to the polar axis Z_4, in the case of equatorial orbits. This phenomenon is known as a precession[18] around Z_4.

The other important fact is that it exists an equilibrium point that vanishes the precession, when the initial condition is $\phi_1 = \phi_2 = 0$ and the tether is rotating inside the equatorial plane.

In contrast, the initial state $\phi_1 = \pm\frac{\pi}{2}$ turns out to be unstable. This means that if the angular momentum is ever contained in the equatorial plane, the tether's attitude would drift away from that angular state.

Another significant case is found when the orbit is polar instead ($i = \pi/2$), leading to an analogous process that yields to a first integral of the motion, that solves to provide the relation

$$\sin\phi_1 \, \cos\phi_2 = \cos\Phi \tag{8}$$

where Φ is a constant that can be determined from the initial values of ϕ_1 and ϕ_2, and again, Φ turns out to be the angle between the tether's angular momentum and the direction perpendicular to the orbital plane (this time a polar orbit).

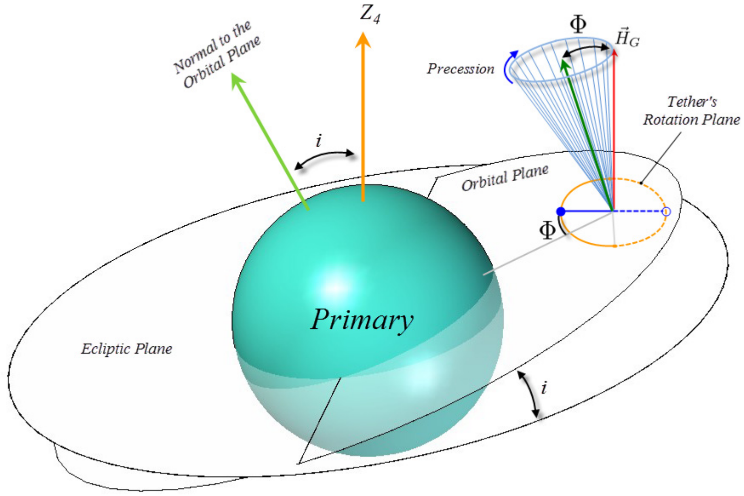


Figure 4: Scheme illustrating the precession of the tether's plane of rotation

The above procedure can be in fact performed particularizing the equations (1) & (2) for an arbitrary value of the orbital inclination, leading to analogous results. Even though the resulting expressions do not provide simple relations like (7) and (8), observation of their numerical solutions confirms that, the precession happens at every arbitrary orbital inclination.

V. Frozen Orbits

A very interesting application or fast rotating tethers, as a consequence of their orbit stabilization capabilities, is the obtention of low eccentricity *frozen orbits*.[8]

Frozen orbits are defined as orbits whose orbital elements remain constant on average. Even though in reality the motion of the spacecraft will not exactly follow the frozen orbits that the averaged model predicts, the long term evolution of the orbit, on average, will actually fit that of the frozen orbit.

The design of frozen orbits has become a key aspect of the mission analysis of lunar probes, trying to prolong their life-time as much as possible by reducing the amount of fuel dedicated to station keeping.[4–6] Scientific space missions usually require high inclination circular orbits, which in the case of the Moon are highly unstable due to the Earth's third body perturbation.[18] Therefore, the search for high inclination and low eccentricity frozen orbits has become a reigning research topic for the last years. We have noticed that rotating space tethers can make a significant contribution to this subject.

Lagrange planetary equations govern the evolution of the orbital elements. When we enter in these equations the averaged gravitational perturbing potential (5) and the averaged third body perturbing potential (6)

$$\langle \mathcal{R} \rangle = \langle \mathcal{R}_g \rangle + \langle \mathcal{R}_\oplus \rangle$$

then the Lagrange equations provide instead the average long term evolution of the orbital elements. According to the definition of frozen orbits, the stationary solutions of this equations system are in fact frozen orbits.

Due to the difficulty of the governing equations of a rotating tether (remember the translational and rotational problems are coupled and must be solved simultaneously), the most advisable way of simplifying the search for frozen orbits is to particularize the attitude of the tether, and assume it rotates fast enough to consider that the attitude remains unchanged for long enough, thus allowing to decouple the translational problem from the rotational one. Of course, this limits the validity of the obtained frozen orbits to a life-time dependent on the ratio $n/\Omega_\perp$, but extending their validity in time would be as simple as increasing the tether's angular velocity. Another alternative, as seen in §IV, would be to choose an attitude of the tether such that it rotates in the orbital plane, which assures the attitude will remain constant.

Hence, we should find an attitude that both simplifies the equations most and provides results of some interest. The most adequate is to choose $\phi_1 = \phi_2 = 0$ as our major study case. So, assuming $\phi_1 = \phi_2 = 0$, the frozen orbits will be given by stationary solutions of the equations system

$$\frac{\mathrm{d}a}{\mathrm{d}t} = 0 \tag{9}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\mu_\oplus}{\rho^3_{\oplus☾}} \frac{e\sqrt{1-e^2}}{n} \frac{15}{8} \sin(2\,\omega)\,\sin^2(i) \tag{10}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = -\frac{\mu_\oplus}{\rho^3_{\oplus☾}} \frac{e^2}{n\sqrt{1-e^2}} \frac{15}{16} \sin(2\,\omega)\,\sin(2\,i) \tag{11}$$

$$\begin{aligned}\frac{\mathrm{d}\omega}{\mathrm{d}t} = {} & \frac{\mu_☾}{n\,a^5\,(1-e^2)^2} \frac{3}{8} \left(5\,\cos^2 i - 1\right) \left[a_2\,L_T^2 + 2\,J_2\,R^2\right] + \\ & + \frac{\mu_\oplus}{\rho^3_{\oplus☾}} \frac{\sqrt{1-e^2}}{n} \frac{3}{4} \left[4\,\cos^2 i + 5\,\cos^2\omega - 5\,\cos^2 i\,\cos^2\omega - 3\right] + \\ & + \frac{\mu_\oplus}{\rho^3_{\oplus☾}} \frac{\cos^2 i}{n\sqrt{1-e^2}} \frac{3}{4} \left[1 + (5\,\sin^2\omega - 1)\,e^2\right]\end{aligned} \tag{12}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{-\cos i}{n\,a^2\sqrt{1-e^2}} \frac{3}{4} \cdot \left[\frac{\mu_☾}{a^3\,(1-e^2)^{\frac{3}{2}}} \left(a_2\,L_T^2 + 2\,J_2\,R^2\right) + \frac{\mu_\oplus}{\rho^3_{\oplus☾}} a^2 \left(1 + (5\,\sin^2\omega - 1)\,e^2\right)\right] \tag{13}$$

The variation of the mean anomaly M is irrelevant, for it does not change the shape nor the orientation of the orbit, and so is not presented here.

This equations set is similar to those obtained by other authors,[1,3,7] that success to include the gravity perturbation due to J_2, but the novelty of our formulation is that eqs (9) to (13) do additionally include the mechanical perturbation due to the rotating tether.

Note the semi-major axis a remains constant for the considered perturbations. This reduces the dimension of our equations system to four.

Additionally, with little algebraic manipulations this set of equations reveals that the polar component of the angular momentum, H, is preserved in the three-times-averaged problem. It is then possible to express the inclination as a function of just the eccentricity and the initial values i_0 and e_0.

$$H = \sqrt{\mu_☾\,a\,(1-e^2)} \cdot \cos(i) = \sqrt{\mu_☾\,a\,(1-e_0^2)} \cdot \cos(i_0)$$

Also note that eq. (13) evidences that a rotating tether increases the regression of nodes. This property of fast rotating tethers had already been discovered in recent researches by our group. In fact, reference 8 explains that a fast rotating tether in a plane parallel to the equatorial plane of the attracting body reinforces the oblateness effect produced by the attracting body.

This result is quite relevant because it is well known that the oblateness perturbation may have a beneficial effect in general scenarios in which other perturbations tend to destabilize the dynamics. Thus, by the simple expedient of lengthening an inert tether, we might mitigate instabilities induced by the dynamics.

This possibility of artificially increasing the effect of the oblateness of a celestial body is of particular interest for the Moon, whose natural oblateness is not as big as for the Earth and consequently the J_2 zonal harmonic does not dominate clearly over all other harmonics.[19,20]

Anyway, note that eqs. (10) to (12) are independent from the motion of the longitude of the ascending node Ω, so we can ignore eq. (13) when determining the frozen orbit solutions. When in a frozen orbit, its value will simply circulate.

Thus, frozen orbits are obtained as solutions of

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\mathrm{d}i}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}t} = 0$$

that yields to a non-linear algebraic equations system on the variables a, e, i and ω.

If we first consider equations (10) and (11) and set them equal to zero, we find the possible solutions

$$e = 0, \qquad i = 0, \qquad \omega = 0 \text{ or } \pi, \qquad \text{and} \qquad \omega = \pm\frac{\pi}{2}$$

Entering each of them (that fixes one of the four variables) into equation (12) results in an implicit equation of three variables that provides the locus of the variables such that the resulting orbit is frozen. In other words, each of the conditions above leads to a different family of frozen orbits, each with different properties.

The condition $i = 0$ leads to the only solution $e = 1$, leaving ω undetermined. Since in this analysis, we are only considering eccentric orbits, this singular solution will not be treated. The condition $e = 0$ is not valid either, since it the argument of periapsis would not be properly defined with this set of orbital elements, so the condition $e = 0$ is discarded as well. This does not mean that circular frozen orbits do not exist, just that they cannot be found unless we use a non-singular orbital elements set instead. Hence, we will just analyze the families $\omega = 0$ or π, and $\omega = \pm\,\pi/2$.

As the frozen orbits are defined for non-tethered satellites by a combination of the four orbital elements a, e, i and ω, where one is fixed by the family, it looks adequate to represent the locus of frozen orbits in a 3D plot whose coordinates are the cited elements. For tethered satellites instead, the length of the tether is a free parameter, so the locus of frozen orbits will be plotted for a fixed value of L_T.

In fact, the capability of the tether to modify the locus of frozen orbits permits to literally *build* or *design* a frozen orbit that best fulfils our mission requirements, just by finding an optimal combination of a, e, i, ω and L_T. Hence, the tether includes a fifth design parameter on which frozen orbits depend, so the tether length can be seen as an extra degree of freedom that the mission analyst could use to design a new frozen orbit or modify an existing one if necessary.

A. Frozen Orbits with $\omega = 0$

Taking the condition $\omega = 0$ as solution for the equations (10) and (11), and entering it into equation (12) results into the implicit equation

$$-(1 - 5\,\cos^2 i)\,(a_2\,L_T^2 + 2\,J_2\,R^2) + 4\,\frac{\mu_\oplus}{\mu_☾}\,\frac{a^5}{\rho_{\oplus☾}^3}\,(1 - e^2)^{\frac{5}{2}} = 0$$

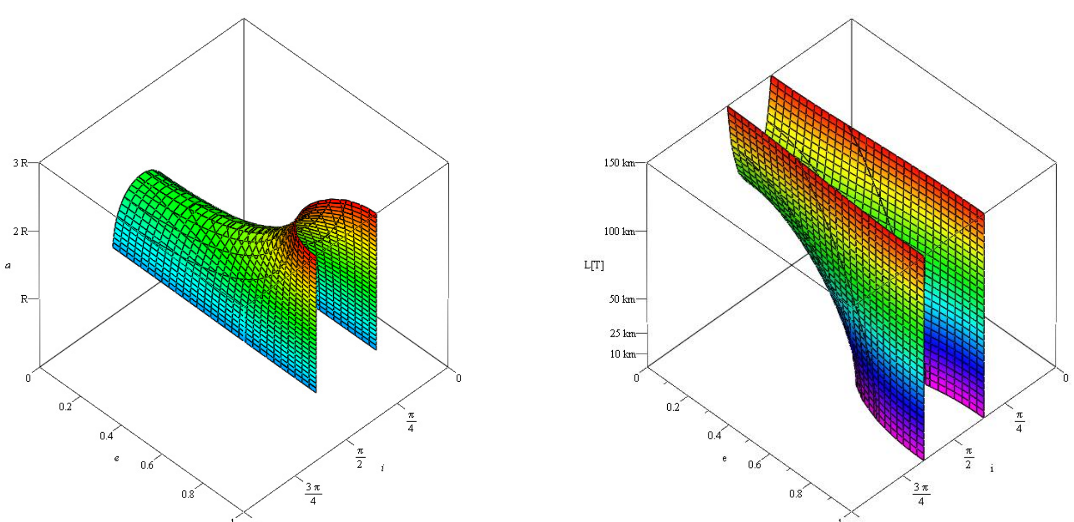


Figure 5: Locus of lunar frozen orbits that fulfil $\omega = 0$. **Left:** Fixed Tether Length **Right:** Fixed Semi-major axis.

This equation reveals that, if we lengthen a tether (increase L_T), in order to keep on satisfying the equality, the locus of frozen orbits will be modified so that I) The range of inclinations where frozen orbits exist will broaden, II) The eccentricities at which frozen orbits exist will diminish, and III) The semi-major axes of existing frozen orbits will augment.

Observing the plots in figure 5 we realize that in this family, frozen orbits exist only for high inclinations (from about 63° to 117°). Low eccentricity orbits exist within the whole range $63° < i < 117°$, but low altitude frozen orbits exist only in the nearby of $i = 65°$ and $i = 115°$; the rest of low eccentricity frozen orbits are attained with high altitude orbits.

The tether has though a positive effect in diminishing the eccentricity of frozen orbits for a given altitude and inclination, while besides extending the range of inclinations for low eccentricity frozen orbits.

B. Frozen Orbits with $\omega = \pi/2$

Taking finally the condition $\omega = \pi/2$ as solution for the equations (10) and (11), and entering it into equation (12) results into the implicit equation

$$-(1 - 5\cos^2 i)\,(a_2 L_T^2 + 2 J_2 R^2) - 2\frac{\mu_\oplus}{\mu_☾}\frac{a^5}{\rho_{\oplus☾}^3}(1-e^2)^{\frac{3}{2}}(3 - 5\cos^2 i - 3e^2) = 0$$

Figure 6 reveals that this family of frozen orbits exist in a narrow range of orbital inclinations, from 40° to 63° (and from 117° to 140°), but for a wide range of eccentricities.

This time as well, the tether enables smaller eccentricities for any given inclination within the range, and slightly increaes the inclination of the frozen orbits. Figure 7 shows as an example the variation over time of the orbital eccentricity and inclination according to a simulation carried out for initial conditions leading to a frozen orbit of this family. The simulation of figure 7 also evidences the importance of the ratio $n/\Omega_\perp$ over long periods of time for cases in which the precession of the rotation plane arises.

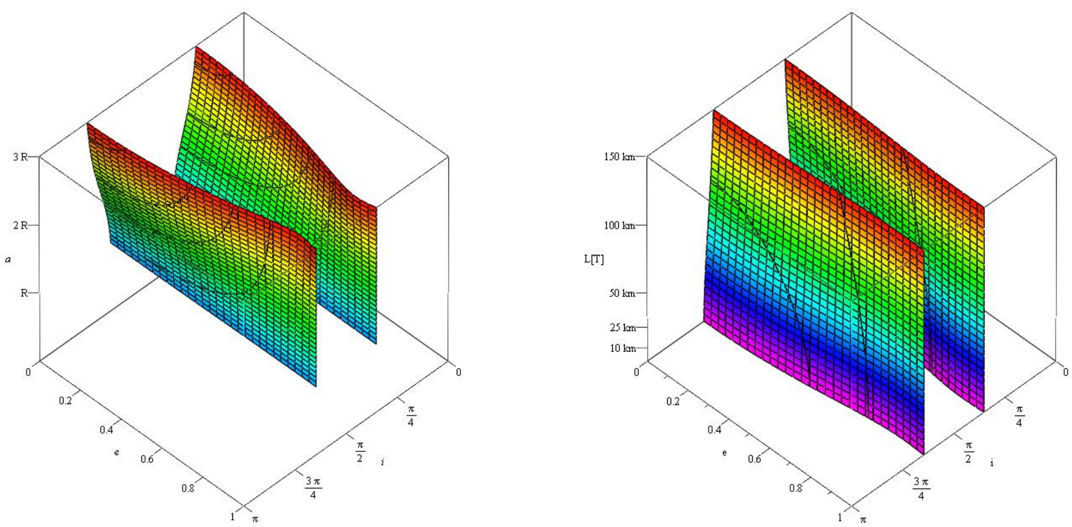


Figure 6: Locus of lunar frozen orbits that fulfil $\omega = \pi/2$. **Left:** Fixed Tether Length **Right:** Fixed Semi-major axis.

VI. Conclusion

Througout this MSc Project[9] we have developed a formulation that allows to study the long term evolution of a tethered system around planetary satellites. This formulation has proved useful to analyze the behaviour of a tether's attitude, as well as applicable for the design of frozen orbits with rotating tethers.

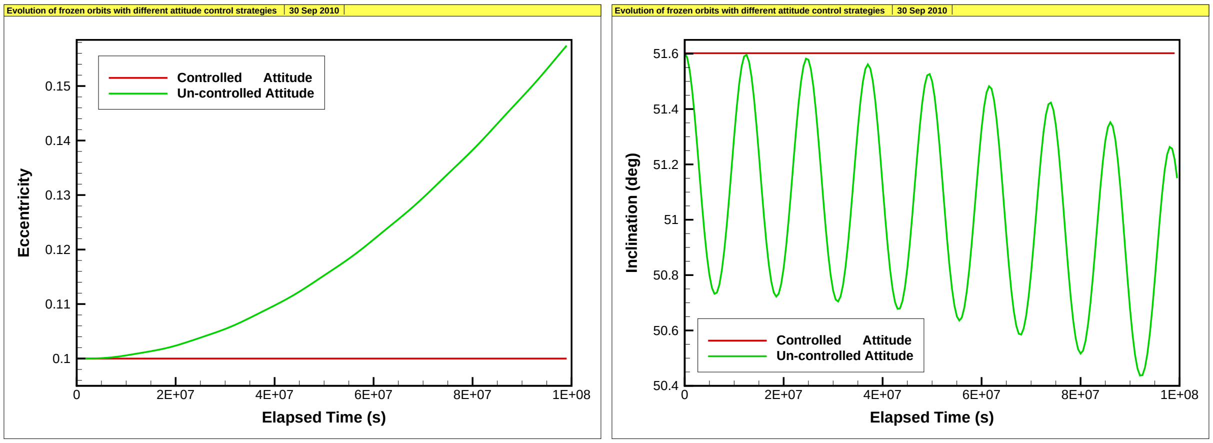


Figure 7: Evolution of a frozen orbit for different control strategies. The red line forces $\phi_1 = \phi_2 = 0$ at any time, while the green line lets the tether's roation plane to precess. The simulation was carried for over 38 months. The nominal frozen orbit is: $\omega = \pi/2$, $e = 0.1$, $a = 2\,R$, $i = 51.601812^\circ$ and $L_T = 40$ km.

With the aid of this formulation, we have concluded that a precession of the rotating tether's angular momentum occurs around the direction perpendicular to the tether's orbital plane. The rate at which the angular momentum precesses is a function of the ratio $n/\Omega_\perp$ and the precession angle Φ formed between the angular momentum vector and the direction vector perpendicular to the orbital plane. Φ is given by the initial conditions $\phi_1(0)$, $\phi_2(0)$ and the orbital inclination i, and remains constant as long as no other outer nor inner torque is applied to the system (so the orbital elements and the rotation speed of the tether do not vary).

We also reason out that there exists an equilibrium attitude that vanishes the precession, which happens when the tether's plane of rotation and the orbital plane are coplanar. Oppositely, when the tether's plane of rotation lies perpendicular to the orbital plane, the tether's attitude is prooved to be unstable and will tend to drift out of that attitude.

By the simple observation of the three-times averaged equations of the motion we were able to reproduce recently discovered orbit stabilization capabilities that arise from the simple fact of lengthening an inert tether, which allows to mitigate instabilities induced by the orbital dynamics in highly perturbed scenarios. This possibility of artificially increasing the effect of the oblateness of a celestial body brings new possibilities to the explorations of celestial bodies, such as the Moon.

Additionally, we shed some light onto the promising capability of rotating tethers to modify frozen orbits, achieving lower eccentricity frozen orbits. In relation to the existence of frozen orbits, the length of the tether becomes an extra parameters in the design of frozen orbits, giving the mission analyst an extra degree of freedom that allows an optimal mission design, by properly selecting the best combination of the orbital elements that leads to the desired frozen orbit.

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