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High fidelity models for orbit propagation: DROMO vs. Störmer-Cowell

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INTRODUCTION

THE simplest analytical model to be used in the propagation of an orbit is the theory of the Keplerian motion of a celestial body. This theory, which is essential from several points of view, becomes hardly useful when perturbations are involved in the dynamics in the sense that analytical solutions are no longer available. There is a large number of perturbations which can be acting on satellites. At first sight and due to the large number of theories and procedures developed in celestial mechanics there would seem to be numerous analytical formulations for solving perturbed motion. Would it be possible to obtain an analytical solution for each particular case? The answer depends on the forces we model; due to the complex nature of the equations representing the physical models exactly integrable expressions are difficult to obtain. The search of analytical solutions usually relies on series expansions to express the motion (a source of practical difficulty). In some cases, the effects over time of given perturbations can be classified through the secular, short-periodic, and long-periodic variations of the classical orbital elements. Although these distinctions help us decide which effects to model, the practical difficulty of an infinite series still remains.

A classical approach to the many-body problem is that of using *special perturbations*. Nowadays and due to the availability of high-speed computers is an essential tool in Space Dynamics which exhibits a great advantage: it is applicable to any orbit involving any number of bodies and all sorts of astrodynamical problems, especially when these problems fall into regions in which general perturbation theories are absent. One such case is, for example, that NEO's dynamics. The main disadvantage of this method is that it rarely leads to any general formulae; in addition, the body's positions at all intermediate steps must be computed in order to arrive at the final configuration. Some classical special perturbation methods are: 1) the Cowell's method and 2) the Encke's method.

In the Cowell's method the equations are formulated in rectangular coordinates and integrated numerically. The Encke's method makes use of the fact that to a first approximation the orbit is a conic section. The integration gives the difference between the real coordinates and the coordinates of the osculating orbit. As time goes on the differences grow, until it becomes necessary to derive a new osculating orbit. This process is called rectification of the orbit.

There is some confusion in the terminology regularly used when describing the numerical propagation of orbits. Some authors talk about the Cowell's method as a special perturbation method; other authors talk about the Cowell's method as a set of multistep algorithms especially designed for the direct integration of second-order differential equations. This situation is probably due to a particular integration scheme called the Störmer-Cowell method which, at present, is widely used to the propagation of orbits in many astrodynamical problems. In the Störmer-Cowell method the Cowell's method (as special perturbation method) is used, by integrating the equations of motion with the Störmer-Cowell formulas. Many people prefer these methods for improved round-off error and ease of programming. But this is an open question and there is no general agreement about the supremacy of any particular method relative to others.

The Group of Tether Dynamics of UPM (GDT) has developed a new regularization scheme —that we call DROMO— which is characterized by only 8 ODE. The basic theory of DROMO can be found in [7]. The method was presented for the first time in the winter meeting of the AAS/AIAA [5].

The novel Special Perturbation Method, called DROMO, is especially appropriated to carry out the propagation of complex orbits, like, for example, NEO's orbits. The main characteristics of the procedure are:

- Unique formulation for the three types of orbits: elliptic, parabolic and hyperbolic. So, the singularity that appears in the proximity of parabolic motion when using different formulations for elliptic and hyperbolic orbits disappears.
- It uses orbital elements as generalized coordinates (as the Lagrange's Planetary equations); as consequence, the truncation error vanishes in the unperturbed problem and is scaled by the perturbation itself in the perturbed one. The method doesn't have singularities for small inclination and/or small eccentricities, unlike the Lagrange's planetary equations. The orbital plane attitude is determined by the Euler parameters which are free of singularities.
- The use of Euler parameters gives easy auto-correction as well as robustness. The error propagation shows better performances than in the cases of Cowell's or Encke's methods. Easy programming, since they use the components of perturbation forces in the orbital frame. This makes easy the use of models proper of Orbital Dynamics.
- A precise and fast simulator is obtained by using this method with variable step routines with effective step control, as Runge-Kutta-Fehlberg or Dormand-Prince types. However, routines with fixed step also can de used without reduction in performances.
- It is not necessary to solve Kepler's equation in the elliptic case, nor the equivalent for hyperbolic and parabolic cases, since time is one of the dependent variables determined by the method itself.

The goal of this communication is to compare the characteristics of DROMO as orbit propagator with the propagation scheme based in the Cowell method by using the Störmer-Cowell algorithms to integrate the equations. In this paper we perform a comparison between two hight fidelity models by using an analytical solution which appears in the well known *problem of Tsien*: a satellite perturbed by a constant radial thrust.

THE TSIEN PROBLEM

A satellite in a circular orbit of radius R_0—circular velocity $R_0\omega_0$ with $\omega_0^2 = \mu/R_0^3$— is acted upon by a constant radial thrust $\vec{\boldsymbol{a}}_p = a_R\,\vec{\boldsymbol{u}}_r$ starting at $t = 0$. Depending of the intensity of the radial thrust a_R two behaviors can be detected. If a_R is greater than a critical value, then an non-Keplerian escape trajectory takes place; otherwise, the radial motion of the S/C keeps bounded.

Classical analysis

The forces are central; therefore the angular momentum is constant and the trajectory is a plane curve:

$$\vec{\boldsymbol{h}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{v}} = \vec{\boldsymbol{r}}_0 \times \vec{\boldsymbol{v}}_0 = R_0^2\omega_0(-\vec{\boldsymbol{j}})$$

Let (r, θ) polar coordinates inside the orbital plane. The law of areas takes the form

$$r^2\,\dot{\theta} = h, \qquad \text{where} \quad h = R_0^2\omega_0 \tag{1}$$

The whole forces acting upon the satellite are conservatives and they derive from the potential energy

$$V(r) = -\frac{\mu}{r} - a_R\,r$$

as a consequence, the total energy is conserved

$$\frac{1}{2}v^2 + V(r) = E, \qquad \text{where} \quad E = \frac{1}{2}v_0^2 - \frac{\mu}{R_0} - a_R\,R_0$$

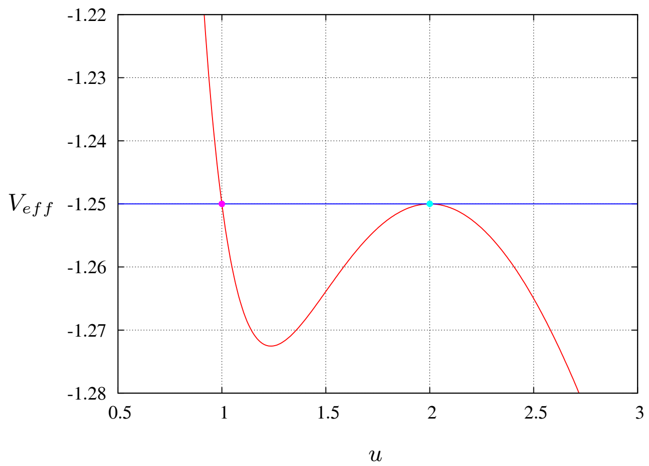

Figure 1. Effective potential V_{eff} and total energy $\mathcal{E}$ for $\epsilon = 1.0$

We introduce the following non-dimensional variables:

$$r = u\,R_0, \qquad \tau = \omega_0\,t, \qquad \epsilon = \frac{8\,a_R}{R_0\omega_0^2}$$

Taking into account the law of areas (1), the energy equation governs the motion relative to the radius vector; it takes the form

$$\frac{du}{d\tau} = \pm\sqrt{\mathcal{E} - V_{eff}(u)}$$

where the effective potential V_{eff} and the total energy $\mathcal{E}$ (non-dimensional values) are given by:

$$V_{eff}(u) = \frac{1}{u^2} - \frac{2}{u} - \frac{\epsilon}{4}u, \qquad \mathcal{E} = -(1 + \frac{\epsilon}{4})$$

The solution is given by the following quadrature:

$$\tau = \pm \int_1^u \frac{d\xi}{\sqrt{\mathcal{E} - V_{eff}(\xi)}} \tag{2}$$

and the motion takes place in regions where

$$\mathcal{E} - V_{eff}(u) > 0$$

Depending on ϵ, two different behaviors appear:

1. $\epsilon < 1$ the thrust is *small* and the motion is bounded by two concentric circles
2. $\epsilon > 1$ the thrust is *large* and the motion is unbounded. In particular, the escape velocity is reached after a while (see details in [1])

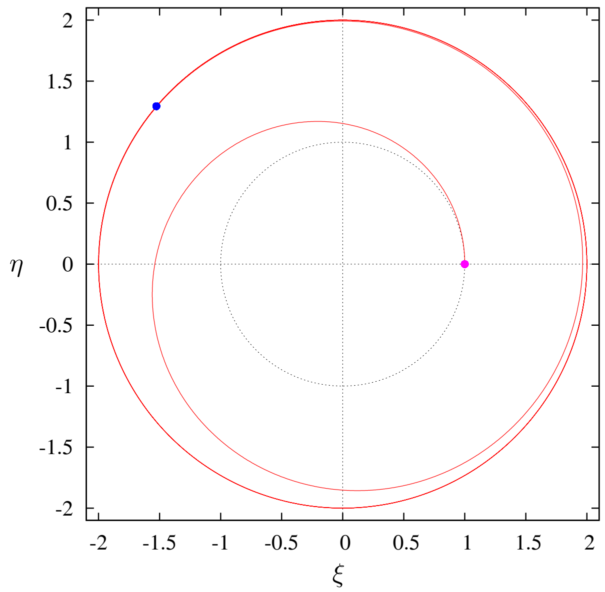

Figure 2: Satellite trajectory in the asymptotic case $\epsilon = 1$

Test solution

There is an asymptotic motion which separates these two different behaviors; it appears for $\epsilon = 1$. In such a case, the energy line $\mathcal{E}$ is tangent to the graphic of the effective potential (see Fig. 1) in a relative maximum which takes place in $u = 2$. In this particular case, equation (2) provides the following solution:

$$\tau = \int_1^u \frac{2\xi d\xi}{(2-\xi)\sqrt{\xi - 1}} \quad \Rightarrow \quad \tau = 4\ln\left[\frac{1+\sqrt{u-1}}{1-\sqrt{u-1}}\right] - 4\sqrt{u-1} \tag{3}$$

Notice that the motion is tending to a circular motion along a circumference of radius $2R_0$ (see Fig. 2).

The *numerical* obtention of this analytical solution is not easy. In effect, the errors accumulated in the calculation prevent the numerical solution to reach the asymptotic behavior for moderately large values of the time τ. These errors *move the energy line* which is no longer tangent to the graphic of the effective potential and 1) the satellite descends towards the starting circle or 2) it escapes from the attractive body. Thus, and due to its well defined analytical solution, the Tsien problem is an excellent tool to compare performances of different propagators and integrators.

The goal is to compare different special perturbation methods (DROMO and Störmer-Cowell) observing the accuracy and the computation time associated with the numerical description of the solution given by (3).

COWELL'S FORMULATION

The *Cowell's formulation* [11] is the name given to a formulation of the satellite's dynamics written down as a system of second-order differential equations for the position coordinates of the satellite. The satellite's motion is governed by

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} + \boldsymbol{a}_p \tag{4}$$

where $\boldsymbol{r}$ is the position vector of the satellite and $\boldsymbol{a}_p$ is the total perturbing acceleration. In the Tsien problem selected as test these governing equations, in non-dimensional variables, take the form

$$\frac{d^2\xi}{d\tau^2} = -\frac{\xi}{\rho^3}\left(1 - \frac{\epsilon}{8}\rho^2\right), \quad \rho = \sqrt{\xi^2 + \eta^2} \tag{5}$$

$$\frac{d^2\eta}{d\tau^2} = -\frac{\eta}{\rho^3}\left(1 - \frac{\epsilon}{8}\rho^2\right) \tag{6}$$

and they must be integrated from the initial conditions:

$$\tau = 0: \quad \xi = 1, \quad \eta = 0, \quad \dot{\xi} = 0, \quad \dot{\eta} = 1 \tag{7}$$

Formulation	Störmer methods	Cowell methods
Non-summed	$\mathbf{r}_{n+1} = 2\,\mathbf{r}_n - \mathbf{r}_{n+1} + h^2 \sum_{j=0}^{k-1} \delta_j \, \nabla^j \mathbf{a}_n$	$\mathbf{r}_{n+1} = 2\,\mathbf{r}_n - \mathbf{r}_{n+1} + h^2 \sum_{j=0}^{k-1} \delta^*_j \, \nabla^j \mathbf{a}_{n+1}$
Summed	$\mathbf{r}_{n+1} = h^2 \sum_{j=0}^{k+1} \delta_j \, \nabla^{j-2} \mathbf{a}_n$	$\mathbf{r}_{n+1} = h^2 \sum_{j=0}^{k+1} \delta^*_j \, \nabla^{j-2} \mathbf{a}_{n+1}$

Table 1: Some of the different formulations available for fixed-stepsize Störmer-Cowell methods[2].

The second-order differential equation (5-6) can be integrated by reducing it to a first order system (which allows the use of a broader class of integration methods); however, it seems more natural to directly integrate Eq. (5-6) without using first derivatives. This approach results in an increase in efficiency [9] because it exploits special information about the differential equations. So, we shall distinguish between double-integration methods that directly integrate the second-order differential equations (5-6) and single-integration methods that integrate first-order differential equations.

Double-integration methods are generally more accurate than single-integration methods, because removing the velocity calculation reduces the round-off error. In addition, in the case of multi-step integration, double-integration methods are more stable than single-integration methods and only require one evaluation per step, so double-integration is faster than single-integration [3].

Runge Kutta Nyström methods (RKN) are single-step double-integrations methods. These methods allow for an easy stepsize control and are well suited for high accuracy requirements. The corresponding multi-step methods are the explicit Störmer methods and the implicit Cowell methods, which are usually combined together in a *predictor-corrector* construction as Störmer-Cowell methods (SC). Störmer-Cowell methods are known to obtain the maximum profit out of the Cowell's formulation, and are hence the ones on which we will focus. Multi-step integrators can be implemented following different formulations. For example they have both a non-summed and a summed form, depending on whether a summation term is used in the derivation[3] (see table 1). The summed form of the Störmer-Cowell method is also known as Gauss-Jackson integration, and is usually preferred since it manages to reduce the round-off error[6].

The formulation of Störmer-Cowell methods is easy for fixed-stepsize —see table 1— and is most clearly expressed in terms of backward differences of the backpoints, but backward differences require that the backpoints are equally spaced, so the stepsize must remain constant[3]. However, it is advantageous for an integrator to be suitable for a variable stepsize formulation, which is not easy in a multi-step method, since it is becomes necessary to recompute the coefficients via recurrences in order to avoid the evaluation of the two-fold integrals that define the coefficients. This is achieved by using divided differences instead of backward differences, for divided differences do not require the backpoints to be equally spaced[9, 2]. This yields the problem of stepsize control, which is usually solved by taking the difference of correctors of different orders to estimate the local error at each step, and the size of the next step is then adjusted based on the local error estimate to meet a given tolerance.

Orbit propagators that implement a Cowell's formulation with a variable-stepsize Störmer-Cowell integration method are believed to provide the best combination of performances in terms of accuracy and speed. However, the implementation of such codes is delicate and non-trivial, since Störmer-Cowell methods might be implemented following many different algorithms, and there are several issues such as the stepsize control strategy or the starting procedure that are very tricky and susceptible of different approaches that lead to different performances.

In order to provide the most fair comparison of propagators we decided to use in this article a well-known, referenced and tested Störmer-Cowell method for performing numerical comparisons. So, we used the code that Matthew M. Berry derived and kindly put freely available[2], where just slight adjustments were made to the code to allow arbitrary order and dimension of the system of differential equations, and a stepsize control based on relative tolerance instead of absolute tolerance. The main features of this code are the following:

- ◇ The method is variable-step with error control, so larger stepsizes can be taken when possible
- ◇ The step size is controlled by estimating the local position error at each step.
- ◇ Only one evaluation is performed per step, for a PEC implementation, which significantly reduces the run-time, and because order increases that require a constant step are not being considered, fewer restrictions are placed on the stepsize control, while preserving the stability of the scheme.
- ◇ The method uses a variable-order implementation for initialization, so it is self-starting. However, the method is not variable-order beyond the initialization phase, because variable-order algorithms would require a second evaluation.

Notice that (unlike DROMO formulation) Cowell's formulation integrated with Störmer-Cowell integrators just provides the propagated position vector but not the velocity. If the velocity is also desired, then the Störmer-Cowell integrator must be combined with an embedded Adams integrator, which usually slightly increases the number of integration steps, as the stepsize required by Adams integrators might be more limiting than that required by Störmer-Cowell integrators.

DROMO

The DROMO project is based in a new special perturbation method whose theory is developed in papers [5, 7] where a set of non-classical elements $(q_1, q_2, q_3, \varepsilon_1^0, \varepsilon_2^0, \varepsilon_3^0, \varepsilon_4^0)$ has been introduced. DROMO provides the time evolution of these elements when the perturbations forces are known.

Starting from the original variable, an slight improvement of the performances of DROMO can be obtained by carrying out the following change of variables:

$$\zeta_1 = \frac{q_1}{q_3}, \qquad \zeta_2 = \frac{q_2}{q_3}, \qquad \zeta_3 = q_3$$

This way the eccentricity vector can be expressed like

$$\vec{\boldsymbol{e}} = \zeta_1\, \vec{\boldsymbol{u}}_1 + \zeta_2\, \vec{\boldsymbol{u}}_2$$

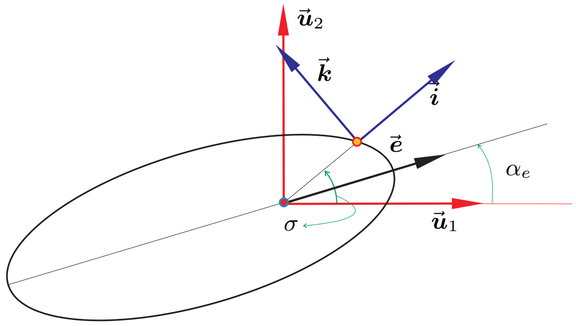

Figure 3: Reference frames

where the unit vectors $(\vec{\boldsymbol{u}}_1, \vec{\boldsymbol{u}}_2)$, which lie in the orbital plane, are defined by:

$$[\vec{\boldsymbol{u}}_1, \vec{\boldsymbol{u}}_2] = [\vec{\boldsymbol{i}}, \vec{\boldsymbol{k}}]\, Q_0, \qquad Q_0 = \begin{pmatrix} \cos\sigma & \sin\sigma \\ -\sin\sigma & \cos\sigma \end{pmatrix}$$

They rotate with angular velocity $+\dot{\sigma}\,\vec{\boldsymbol{j}}$ relative to the orbital frame $(\vec{\boldsymbol{i}}, \vec{\boldsymbol{k}})$.

Expressed in terms of these new variables the governing equations take the form,

$$\begin{aligned}
\frac{d\zeta_1}{d\sigma} &= \frac{1}{\zeta_3^4 \hat{s}^3}\left[+\hat{s}\sin\sigma f_{px} + \{\zeta_1 + (1+\hat{s})\cos\sigma\} f_{pz}\right] \\
\frac{d\zeta_2}{d\sigma} &= \frac{1}{\zeta_3^4 \hat{s}^3}\left[-\hat{s}\cos\sigma f_{px} + \{\zeta_2 + (1+\hat{s})\sin\sigma\} f_{pz}\right] \\
\frac{d\zeta_3}{d\sigma} &= -\frac{1}{\zeta_3^3 \hat{s}^3} f_{pz} \\
\frac{d\tau}{d\sigma} &= \frac{1}{\zeta_3^3 \hat{s}^2} \\
\frac{d\varepsilon_1^0}{d\sigma} &= -\frac{\lambda(\sigma)}{2}\{\sin(\sigma-\sigma_0)\varepsilon_2^0 + \cos(\sigma-\sigma_0)\varepsilon_4^0\} \\
\frac{d\varepsilon_2^0}{d\sigma} &= +\frac{\lambda(\sigma)}{2}\{\sin(\sigma-\sigma_0)\varepsilon_1^0 - \cos(\sigma-\sigma_0)\varepsilon_3^0\} \\
\frac{d\varepsilon_3^0}{d\sigma} &= +\frac{\lambda(\sigma)}{2}\{\cos(\sigma-\sigma_0)\varepsilon_2^0 - \sin(\sigma-\sigma_0)\varepsilon_4^0\} \\
\frac{d\varepsilon_4^0}{d\sigma} &= +\frac{\lambda(\sigma)}{2}\{\cos(\sigma-\sigma_0)\varepsilon_1^0 + \sin(\sigma-\sigma_0)\varepsilon_3^0\}
\end{aligned}$$

These equations should be integrated, taking into account the relations:

$$\begin{aligned}
\lambda(\sigma) &= \frac{1}{\zeta_3^4\, \hat{s}^3} f_{py} \\
\hat{s} &= 1 + \zeta_1\cos\sigma + \zeta_2\sin\sigma \\
z &= \frac{1}{r} = \zeta_3^2\,\{1 + \zeta_1\cos\sigma + \zeta_2\sin\sigma\} \\
\frac{dr}{d\tau} &= \zeta_3(\zeta_1\sin\sigma - \zeta_2\cos\sigma) \\
\chi &= \frac{\sigma-\sigma_0}{2}
\end{aligned}$$

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_3 \\ \varepsilon_2 \\ \varepsilon_4 \end{pmatrix} = \begin{pmatrix} \cos\chi & \sin\chi & 0 & 0 \\ -\sin\chi & \cos\chi & 0 & 0 \\ 0 & 0 & \cos\chi & -\sin\chi \\ 0 & 0 & \sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} \varepsilon_1^0 \\ \varepsilon_3^0 \\ \varepsilon_2^0 \\ \varepsilon_4^0 \end{pmatrix}$$

and starting from the appropriate initial conditions at $\sigma = \sigma_0$ $(\tau = 0)$. Here (f_{px}, f_{py}, f_{pz}) are the non-dimensional components of the perturbing force acting upon the satellite.

For the Tsien problem solved in these pages $(f_{px} = \epsilon/8,\, f_{py} = f_{pz} = 0)$ and the orbital plane remains constant; as a consequence, the Euler parameters $(\varepsilon_1^0, \varepsilon_2^0, \varepsilon_3^0, \varepsilon_4^0)$ remains constant and only the equations for the variables $(\tau, \zeta_1, \zeta_2, \zeta_3)$ change with time. The unit vectors $(\vec{\boldsymbol{u}}_1, \vec{\boldsymbol{u}}_2)$ are fixed in the inertial space and the initial conditions are:

$$\sigma = \sigma_0 = 0: \quad \tau = 0, \quad \zeta_1 = 0, \quad \zeta_2 = 0, \quad \zeta_3 = 1 \tag{8}$$

The translation from the DROMO-elements to coordinates are given by:

$$\xi = \frac{1}{\zeta_3^2 \hat{s}}\cos\sigma, \qquad \dot{\xi} = -\zeta_3(\zeta_2 + \sin\sigma)$$

$$\eta = \frac{1}{\zeta_3^2 \hat{s}}\sin\sigma, \qquad \dot{\eta} = +\zeta_3(\zeta_1 + \cos\sigma)$$

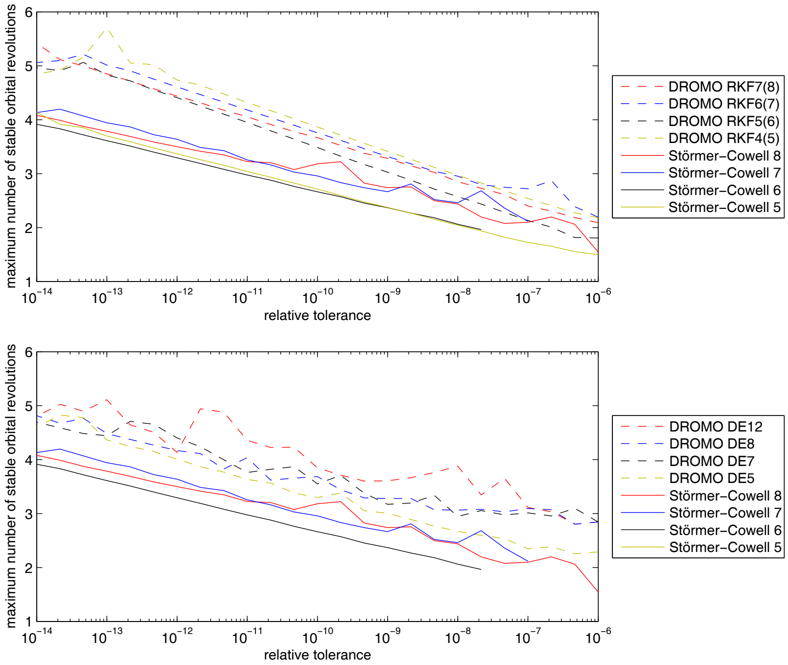

Figure 4: Comparison of method stability versus relative tolerance

COMPARISON

The asymptotic orbit, which should be calculated numerically, is very unstable and it can be assumed that any propagator will only be able to obtain a stable solution for a few orbits. *It is clear that a more accurate integration scheme permits to describe the asymptotic orbit during a longer time.* In this chapter the stability of the presented methods and the computational cost will be analyzed and compared.

Stability

A suitable measure to evaluate performance of the presented propagators is to calculate the number of orbits until the numerical solution starts to deviate from the asymptotic orbit. A deviation is considered, when the relative error of the numerically computed position is larger than a threshold. Here R is the current orbital radius which must be compared with the radius of the asymptotic orbit $2R_0$.

$$\frac{|2R_0 - R|}{2R_0} < 10^{-3}$$

To allow for a fair comparison, integrators of same order are used for the DROMO and Störmer-Cowell formulations. For DROMO, the integrators of the Runge-Kutta-Fehlberg family have proven to be very efficient and accurate. These schemes [8, 4] of order 5 to 8 are compared to Störmer-Cowell implementations [2] of equal maximum order. In addition, integrators of the multistep method of Shampine & Gordon [10] (DE 5-8) are tested and compared to Störmer-Cowell, too. The implementations for Störmer-Cowell and the DE integrators are modified to obtain a fixed order version to be compared with RKF integrators.

Figure 4 shows the number of stable orbits based on the initially given relative tolerance of the integrators. It is evident that DROMO in combination with RKF integrators has a better stability than Störmer-Cowell. However the runtime of the DROMO method is higher than of the Störmer-Cowell method of equal order. This drawback can in part be accounted for by using the DE integrator, which is faster but less accurate, for DROMO.

Method	DROMO RKF7(8)	DROMO RKF6(7)	DROMO DE8	DROMO DE7	SC 8th order	SC 7th order
Rel tolerance	1e-11	1e-11	1e-11	1e-12	1e-14	1e-14
Runtime in s	0.21	0.47	-	-	0.24	0.24
Function calls	2379	4850	1113	1623	439	536
Number of steps	181	483	-	-	431	529

Table 2: Runtime comparison for 4 complete orbits

Computational Cost

In order to evaluate the computational cost of the different methods under equal conditions, the integrators have to be tuned to a similar performance. Therefore a common integration range and accuracy is chosen for them. According to figure 4, all integrators can be stable for up to 4 orbital revolutions. For fair comparison, the relative errors are chosen in such a way, that the integrators are stable only within that specified range. DROMO RKF7(8) and RKF6(7) can achieve this with $\epsilon_{rel} = 10^{-11}$ while the equal order Störmer-Cowell propagators need a tighter tolerance of $\epsilon_{rel} = 10^{-14}$. The results show comparison only of integrators of order 7 and 8 because, for both methods, they perform significantly better in terms of runtime. The evaluation is performed 100 times and table 2 shows the mean runtime, the number of steps and the function calls. It indicates similar processing time for DROMO RKF and Störmer-Cowell of the same order even though the number of function calls of DROMO is higher. This is due to the specific characteristics of the Tsien problem. The higher number of function calls in DROMO does not influence the runtime significantly, because the calculation of the perturbations is not very costly. In the Störmer-Cowell method the runtime is influenced by the fact, that coefficients have to be recalculated for each integration step. Using DROMO formulation in combination with the multistep DE integrator requires less function calls. For these integrators the runtime are not shown because they are implemented in a different programming environment.

CONCLUSIONS

From the analysis carried out in this paper some conclusions can be drawn.

- In terms of *accuracy* DROMO with the Runge-Kutta-Fehlberg routine RKF7(8) turn out to be the best combination since they provide a longer and more stable description of the asymptotic orbit.
- In terms of *function calls* the Störmer-Cowell formulations turns out to be the best formulation since it provides the lower number of call to the derivative functions.

Due to the plus of accuracy provided by the DROMO formulation, this scheme is the most appropriated for the propagation of orbits when a high-fidelity description of the trajectory is mandatory. This plus of accuracy, however, has a cost: the higher number of function calls due to the Runge-Kutta-Fehlberg routine used to perform the integration.

However, and from a *global point of view*, the combination of DROMO with the multistep method of Shampine & Gordon [10] (DE) shows excellent characteristics because: 1) the accuracy worsens in a small amount, relative to the accuracy provided by the combination DROMO + RKF7(8), and 2) the number of function calls reduce in a significant way. Regarding this last point, it should be noticed that the Störmer-Cowell formulas requires one function call per step, and the multistep method of Shampine & Gordon [10] (DE) requires two function call per step due to the second evaluation that takes place in the *correction* part of the algorithm.

The runtime is not a reliable parameter because: 1) is influenced by the MATLAB environment in which most of the calculations have been made, and 2) the simplicity of the derivatives in the Tsien problem used to test the different schemes leads to an almost zero computational cost which cannot be extrapolated to the propagation of real orbits.

In any case, this paper is a first approximation that would be duly qualified in the future.

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REFERENCES

[1] Richard H. Battin. *An Introduction to the Mathematics and Methods of Astrodynamics*. Educational Series. AIAA, revised edition, 1999.

[2] Matthew M. Berry. *A Variable-Step Double-Integration Multi-Step Integrator*. PhD thesis, Virginia Tech, Blacksburg, 2004.

[3] Matthew M. Berry and Liam M. Healy. Speed and accuracy tests of the variable-step störmer-cowell integrator. In *15th Annual AAS/AIAA Spaceflight Mechanics Meeting*. AAS/AIAA, January 2005.

[4] Erwin Fehlberg. Classical fifth-, sixth-, seventh-, eight-order runge-kutta formulas with stepsize control. Technical Report NASA TR R-287, George C. Marshall Space Flight Center, 1968.

[5] J. M. Hedo J. Peláez and P. Rodríguez de Andrés. A special perturbation method in orbital dynamics. Paper AAS 05-167 of the 15th AAS/AIAA Space Flight Mechanics Meeting, Copper Mountain, Colorado, USA, 23-27 January 2005, 2005.

[6] Oliver Montenbruck and Eberhard Gill. *Satellite Orbits - Models, Methods and Applications*. Springer, 1st edition, 2005.

[7] Jesús Peláez, José Manuel Hedo, and Pedro Rodriguez de Andrés. A special perturbation method in orbital dynamics. *Celestial Mechanics and Dynamical Astronomy*, 97:131–150, 2007. 10.1007/s10569-006-9056-3.

[8] William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery. *Numerical Recipes - The Art of Scientific Computing*. Cambridge University Press, third edition, 2007.

[9] H. Ramos and J. Vigo-Aguiar. Variable stepsize störmer-cowell methods. *Mathematical and Computer Modelling*, 42(7-8):837 – 846, 2005. International Conference of Computational Methods in Sciences and Engineering 2003.

[10] L. F. Shampine and M. K. Gordon. Local error and variable order adams codes. *Applied Mathematics and Computation*, 1:47–66, 1975.

[11] David A. Vallado. *Fundamentals of Astrodynamics and Applications*, volume 21 of *Space Technology Library*. Microcosm Press and Springer, 3rd edition, 2007.