Monografías de la Real Academia de Ciencias de Zaragoza. **xxx: ??–??**, (2009).

Plasmasphere Sailing with Passive Electrodynamic Tethers

Claudio Bombardelli, Jesús Peláez Álvarez y Manuel Sanjurjo Rivo

Grupo de Dinámica de Tethers (GDT).

Dept. Física Aplicada a la Ingeniería Aeronáutica

Universidad Politécnica de Madrid, 28040 Madrid, Spain.

Abstract

It was recently shown [1] that an electrodynamic tether operating in passive mode (i.e. without onboard power supplies) and orbiting around a planet with a rigidly corotating plasmaphere can be used to increase the energy of the orbit as long as the orbital eccentricity and/or inclination is not zero. In the case of non-circular equatorial orbits results has shown that a decrease in orbit eccentricity corresponds to an increase in semimajor axis until full circularization. In this article we address the case of inclined orbits showing that in this circumstance one can achieve the highest performance in terms of orbit raising. Orbital energy, which is extracted from the kinetic energy of the planet corotating plasmasphere, is increased at the expense of orbit inclination until an equatorial orbit is reached and thrust is no longer possible. Examples are presented for systems operating in Earth and Jupiter orbit.

1 Introduction

The cost of power generation, both in terms of mass and system complexity, is arguably the biggest obstacle to the development of efficient electric propulsion systems in space. The potentially huge propellant mass savings of high specific impulse electrostatic or electromagnetic thrusters, can be in fact only partially exploited in real space missions due to the cost of on board power production. Solar panels, which are almost always the power generation system of choice in space mission, can presently reach about 10 to 20 kg/kW specific mass in Earth orbit while progress in photovoltaic technology should lead to an 5kg/kW specific mass in the future. In addition, solar panels need to be constantly track the sun direction if high efficiency is desired and do not work during eclipse time. Systems offering higher power density generation capability or simpler control management are therefore highly desirable.

An Electrodynamic tethers (EDT) is a space apparatus which can supply power and/or propulsion to a spacecraft by exploiting the electromagnetic interaction of a conducting cable orbiting around a planet with a magnetic field and reasonable plasma density. As such interaction occurs without the need of expending fuel EDTs can be used as propellantless propulsion systems of great interest in space technology. After its first appearance in the literature [2] the concept of electrodynamic tether has been studied and refined until the introduction of the higher performance bare electrodynamic tether concept in 1991 by Sanmartin et al. [3].

In general an EDT can operate either in *passive mode*, in which case a current flows through the tether without the need of a spacecraft power supply, or in *active mode*, in which the current is driven through the tether at the expense of power generated onboard the spacecraft (e.g. with solar arrays). It is customarily believed that in a passive EDT orbital energy is always dissipated to result in spacecraft deorbiting with the option of having part of that energy stored or directly used as onboard power. This is, however, not always the case and there are examples in the literature

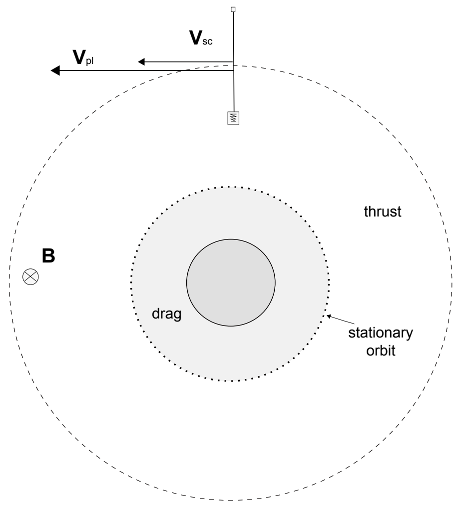

Figure 1: Thrust and drag conditions for an EDT in circular equatorial orbit around Jupiter

that highlight a more complex behavior. The case of EDTs around Jupiter is particularly instructive in this regard. As shown by Sanmartin and Lorenzini ([4]) a passive EDT in a *prograde circular equatorial* orbit experiences thrust beyond the stationary radius ($r_s = 2.24 r_J$ with r_J indicating the equatorial Jupiter radius) where Jupiter-corotating plasma exceeds orbital speed and can be used as a source of thrust *and* power generation. More recently Bombardelli and Pelaez ([1]) have concluded that in the most general case a passive EDT can be used to increase the orbital energy of any orbit with the exception of circular equatorial orbits below the stationary radius. In the same article, an analysis of orbit energy raising starting from equatorial elliptic orbits around Jupiter was performed showing that plasmasphere thrust maneuver resulted in a circularization of the orbit together with an increase in semimajor axis. In this article we investigate the orbit raising capability of passive EDTs starting from inclined orbits.

2 Lorentz Force Curve

When correct contact with the surrounding plasma is established[1] an EDT experiences the Lorentz force:

$$\mathbf{F} = I_{av} L (\mathbf{u}_t \wedge \boldsymbol{B}) \tag{1}$$

where L is the tether length, I_{av} the current averaged over the tether length, $\mathbf{u}_t$ the tether line unit vector and $\boldsymbol{B}$ the local magnetic field.

Following the work of Bombardelli et al. [5] the maximum average current flowing through an EDT with tether under orbital motion limited current collection, can be written, in the hipothesis of small ohmic effects and negligible potential drop at the cathode, as:

$$I_{av} = \frac{3}{5} \eta_\theta I_{ch} \tag{2}$$

where I_{ch} is the characteristic tether current and η_θ is the thrust ohmic efficiency . For a thin

[1] other than bare tether anodic contact this requires a hollow cathod placed at one or both tether ends

tape tether of width w the former is equal to:

$$I_{ch} = \frac{4w}{3\pi} N_e \sqrt{\frac{2E_t}{m_e} q_e^3 L^3}, \tag{3}$$

where N_e is the local plasma electron density, q_e and m_e the electron charge and mass, respectively, and where E_t is the projection of the local motional electric field along the tether line[2]:

$$E_t = |[(\boldsymbol{v}_{sc} - \boldsymbol{v}_{pl}) \wedge \boldsymbol{B}] \cdot \boldsymbol{u}_t| \tag{4}$$

with $\boldsymbol{v}_{sc}$ and $\boldsymbol{v}_{pl}$ indicating the spacecraft and plasma velocity vectors, respectively, and $\boldsymbol{u}_t$ the tether line unit vector.

The thrust ohmic efficiency can be accurately evaluated with the following formula [5]:

$$\eta_\theta \simeq \begin{cases} 1 - \frac{3}{8}\epsilon + \frac{63}{880}\epsilon^2 & \epsilon \leq 1.6 \\ \frac{5}{3\epsilon}\left(1 - \frac{1}{(2\epsilon)^{2/3}}\right) & \epsilon > 1.6 \end{cases}, \tag{5}$$

Where ϵ is the ratio between the tether ohmic impedance and the equivalent plasma impedance:

$$\epsilon = \frac{4}{3\pi}\sqrt{\frac{2q_e^3}{m_e}} \frac{N_e L^{3/2}}{\sigma h E_t^{1/2}}, \tag{6}$$

with σ indicating the tether conductivity and h the tape tether thickness.

Eq. 1 can be compacted in one formula by use of Eq.s (2-5):

$$\boldsymbol{F} = \frac{4\eta_\theta w}{5\pi} N_e B \sqrt{\frac{2q_e^3 E_t L^5}{m_e}} \left(\boldsymbol{u}_t \wedge \boldsymbol{u}_B\right), \tag{7}$$

with $\boldsymbol{u}_v$ and $\boldsymbol{u}_B$ indicating the instantaneous velocity and magnetic field unit vectors, respectively.

The parameter ϵ quantifies the incidence of ohmic effects and depends not only on the position and velocity of the tether along the orbit but also on its attitude. After calling β the angle between the tether line $\boldsymbol{u}_t$ and the local motional electric field $\boldsymbol{E}$ we have:

$$\epsilon = \frac{\epsilon_{min}}{\sqrt{|cos\beta|}}$$

where ϵ_{min} is the value of ϵ obtained when the tether is aligned with the local motional electric field **E** and is defined as:

$$\epsilon_{min} = \epsilon(\beta = 0) = \frac{4}{3\pi}\sqrt{\frac{2q_e^3}{m_e}} \frac{N_e L^{3/2}}{\sigma h E^{1/2}}. \tag{8}$$

Let us now set a right-handed Cartesian reference system centered at the tether center of mass, with the z axis aligned with the local magnetic field vector $\boldsymbol{B}$ and the y axis aligned with local motional electric field vector $\boldsymbol{E}$, which is, by definition, orthogonal to the former.

If we assume the tether line to be constantly orthogonal to the local magnetic field vector the Lorentz force is the highest achievable and its components along the x and y axis are:

$$F_x = F_{max} \times \frac{\eta_\theta(\epsilon)}{\eta_\theta(\epsilon_{min})} cos^2\beta$$

$$F_y = F_{max} \times \frac{\eta_\theta(\epsilon)}{\eta_\theta(\epsilon_{min})} cos\beta sin\beta$$

where

$$F_{max} = \frac{4w\eta_\theta(\epsilon_{min})}{5\pi} N_e B \sqrt{\frac{2q_e^3 E L^5}{m_e}}$$

[2] we assume that two hollow cathodes, one at each tether end, are used

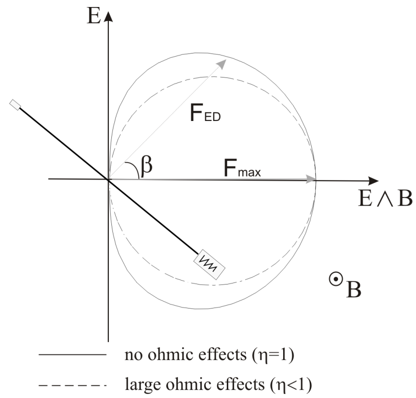

Figure 2: Force curve for a passive EDT as a function of the angle β between the tether line and the local motional electric field. The case of large and negligible ohmic effects are compared.

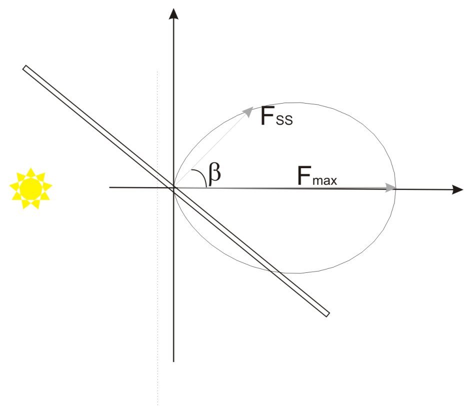

Figure 3: Force curve for a perfectly refelective solar sail as a function of the solar sail "clock angle" β.

The force vector describes a curve in the $x-y$ plane which is plotted in Fig.2. The force curve of a solar sail is added for comparison in Fig.

3 Thrust Attitude Range and Optimum Attitude

Starting from the previously derived Lorenz force curve and neglecting ohmic effects ($\eta_\theta = 1$) let us call $\boldsymbol{v}_{sc}^{\perp}$ and $\boldsymbol{v}_{pl}^{\perp}$ the component of the spacecraft and plasma velocity, respectively, orthogonal to the local magnetic field $\boldsymbol{B}$. Note that, by definition of motional electric field $\boldsymbol{E}$, the component of the relative velocity vector orthogonal to the magnetic field $\triangle\boldsymbol{v}^{\perp} = \boldsymbol{v}_{sc}^{\perp} - \boldsymbol{v}_{pl}^{\perp}$ has to be parallel and opposite to the vector $\boldsymbol{E} \wedge \boldsymbol{B}$ which is equivalent to say that $\boldsymbol{v}_{pl}^{\perp}$ $\boldsymbol{v}_{sc}^{\perp}$ must have the same component along $\boldsymbol{E}$.

After examining Fig. one can see that unless $\boldsymbol{v}_{sc}^{\perp}$ is parallel and opposite to $\boldsymbol{E} \wedge \boldsymbol{B}$ there always exist a range of tether attitudes $\Delta\beta$ for which orbit thrust (in the sense of orbital energy increase) is obtained. After calling ϕ the angle between $\boldsymbol{v}_{sc}^{\perp}$ and the $\boldsymbol{E} \wedge \boldsymbol{B}$ vector the thrust attitude range can be written as:

$$\Delta\beta = \begin{cases} \left[-\frac{\pi}{2}; \phi + \frac{\pi}{2}\right] & -\pi < \phi < 0 \\ \left[\phi - \frac{\pi}{2}; \frac{\pi}{2}\right] & 0 < \phi < \pi \end{cases} \tag{9}$$

In addition, the optimum orientation β_{opt} of the tether line for maximum thrust can be derived as[1]:

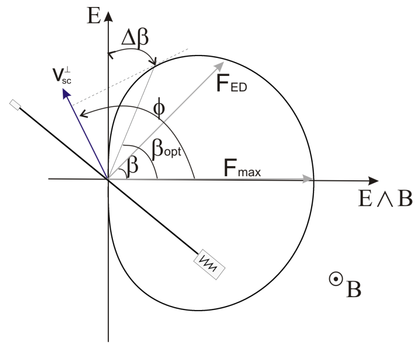

Figure 4: Force curve for a perfectly refelective solar sail as a function of the solar sail "clock angle" β.

$$\beta_{opt} \simeq 2\tan^{-1}\left(\frac{\phi}{\pi}\right) \tag{10}$$

The angle β_{opt}, and, in turns, the angle ϕ should be as small as possible to achieve higher thrust (4). The latter can be developed as:

$$\phi = \cos^{-1}\left[\frac{\boldsymbol{v}_{sc}^{\perp}\cdot\left(\boldsymbol{v}_{pl}^{\perp}-\boldsymbol{v}_{sc}^{\perp}\right)}{\left\|\boldsymbol{v}_{sc}^{\perp}\right\|\left\|\boldsymbol{v}_{pl}^{\perp}-\boldsymbol{v}_{sc}^{\perp}\right\|}\right] \tag{11}$$

When the local plasma velocity exceeds the spacecraft velocity ($\boldsymbol{v}_{sc}^{\perp} < \boldsymbol{v}_{pl}^{\perp}$) the minimum value the angle ϕ can reach is zero and occurs, as it comes from intuition, when the two vectors are parallel. In this case the EDT is sailing "downwind", β_{opt} is zero and the Lorentz force achievable is the maximum possible.

More interesting, and less intuitive, is the case of the EDT crossing a "slower plasmasphere" ($\boldsymbol{v}_{pl}^{\perp} < \boldsymbol{v}_{sc}^{\perp}$). In this case the minimum value for the angle ϕ occurs when the spacecraft velocity component in along the plasma velocity vector $\boldsymbol{v}_{pl}^{\perp}$is equal to the plasma velocity magnitude $\left\|\boldsymbol{v}_{pl}^{\perp}\right\|$ or, equivalently, when the following identity holds [1]:

$$\left(\boldsymbol{v}_{pl}^{\perp}-\boldsymbol{v}_{sc}^{\perp}\right)\cdot\boldsymbol{v}_{pl}^{\perp} = 0 \tag{12}$$

which plugged into Eq. (11) yields:

$$\phi_{min} = \frac{\pi}{2} + \cos^{-1}(\rho), \tag{13}$$

where ρ is the ratio between the component orthogonal to the magnetic field $\boldsymbol{B}$ of the plasma velocity and spacecraft velocity respectively:

$$\rho = \frac{v_{pl}^{\perp}}{v_{sc}^{\perp}} \tag{14}$$

Eq.(13) shows us the role of the plasmasphere velocity in providing thrust to the passive EDT. Note in fact that if the plasma velocity $\boldsymbol{v}_{pl}^{\perp}$ is set to zero the spacecraft velocity $\boldsymbol{v}_{sc}^{\perp}$ is "condemned" to be parallel and opposite to the vector $\boldsymbol{E}\wedge\boldsymbol{B}$ (i.e. $\phi_{min} = 0$) thus making EDT thrust impossible.

From a quantitative point of view we can compute the *thrust reduction coefficient* defined as the ratio between the Lorentz force component along the velocity vector and the maximum force magnitude F_{max}. Assuming the spacecraft velocity is normal to the local magnetic field the latter yields ([1]):

$$\mu_T = \frac{\boldsymbol{F}\cdot\mathbf{v}_{sc}}{F_{max}} = \sqrt{\cos\beta_{opt}}\cos\left(\phi-\beta_{opt}\right) \tag{15}$$

By use of Eq. (10,13,14) we can write the thrust reduction coefficient in power series of ρ:

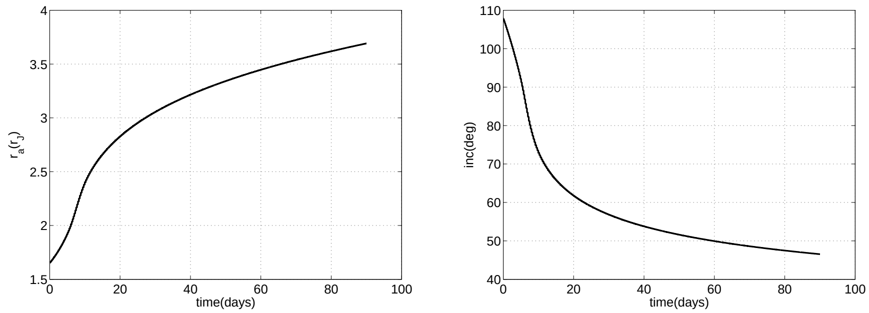

Figure 5: Semimajor axis and inclination variation for an optimaly oriented passive EDT starting from a high inclination ($i_0 = 108^o$) low jovian orbit ($r_{p0} = 1.1 r_J$) with initial eccentricity of 0.2. A 270 kg aluminum tape tether of 40 km length, 5 cm width and 0.05 mm thickness is employed. The payload mass is set to 200 kg.

$$\mu_T = \frac{\pi - 1}{\pi^{5/2}} \rho^{3/2} + \mathcal{O}\left(\rho^{5/2}\right), \tag{16}$$

which highlights the importance of having the highest possible local plasma velocity when compared to the spacecraft velocity when passive EDT thrust is sought. The parameter ρ is ultimately the one gauging the propulsive capability of a planetary plasmasphere. For a comparison, if we consider a circular "grazing" orbit around Earth and Jupiter we obtain about 7.9 km/s and 42.1 km/s of orbital velocities, respectively, while the local plasma velocities for the two cases at the equator are, respectively 0.46 and 12.6 km/s resulting into velocity ratios of 0.058 and 0.3, respectively. This translates into more than one order of magnitude difference in thrust reduction coefficient (Eq.(16)). If we add that the local electric field of Jupiter exceeds the terrestrial one by almost two order of magnitudes and ohmic effects degrade the performance around Earth we conclude that the propulsion capability of Jupiter plasmasphere is far superior.

4 Energy Raising in Inclined Orbits

Optimum spacecraft velocity conditions with respect to the plasma (Eq.(12)) are obtained for orbits which are nearly polar so it makes sense to consider high inclination orbits as starting point for the proposed concepts. The effect of the Lorentz force will tend to decrease the orbit inclination which can be exploited to reach equatorial orbits when needed but will inevitably make the optimum plasmasphere sailing conditions only transient.

Figure 5 displays the semimajor axis and inclination variation for an EDT starting from a high-inclination Jovian orbit. The system is able to exploit the propulsive capability of Jupiter fast-rotating plasmasphere to reach beyond Jove-stationary altitude. It can be seen that highest-efficiency orbit raising occurs around 80 deg inclination. Figure 6 shows how the same EDT can be passively driven from a quasi-equatorial retrograde orbit to a polar orbit.

Performance in Earth orbit is considerably worse for the reasons mentioned above as it can be seen in Figure 7. The orbit energy increase is rarther slow and considerably better performance can be achieve with an active EDT of equal mass including the power subsystem.

5 Conclusions

Passive electrodynamic tethers (EDT) can be used also to increase orbital energy by harnessing the kinetic energy of a planet corotating plasmasphere. The performance in terms of orbit raising is best when high-inclination (quasi-polar) orbits are considered. In the Jupiter case, where plasmasphere

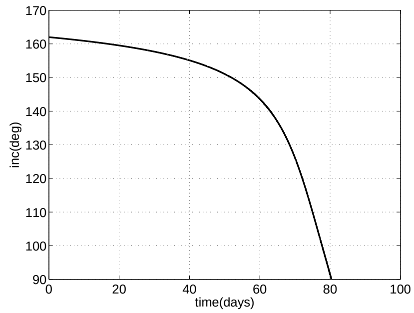

Figure 6: Inclination variation for an optimaly oriented passive EDT starting from a quasi-equatorial retrograde low jovian orbit ($r_{p0} = 1.1 r_J$) with initial eccentricity of 0.2. A 270 kg aluminum tape tether of 40 km length, 5 cm width and 0.05 mm thickness is employed. The payload mass is set to 200 kg.

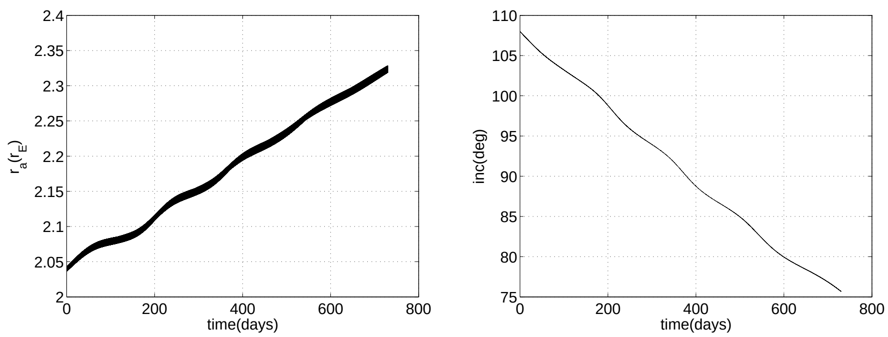

Figure 7: Semimajor axis and inclination variation for an optimaly oriented passive EDT starting from a high inclination ($i_0 = 108^o$) low Earth orbit ($r_{p0} = 1.1 r_E$) with initial eccentricity of 0.2. A 70 kg aluminum tape tether of 10 km length, 5 cm width and 0.05 mm thickness is employed. The payload mass is set to 100 kg.

tangential velocity is already quite high (12 km/s) in low altitude orbits, offers interesting possibilities in terms of orbit raising at the expense of orbit inclination showing that the passive EDT can reach Io orbit in about 3 months starting from a low quasi-circular Jovian orbit. In Earth orbit the much slower rotating plasmasphere combined with the weaker electric field and performance-degrading ohmic effects make the concept considerably less interesting.

6 Acknowledgments

The work was conducted in the framework of the research project “Propagation of Orbits, Advanced Orbital Dynamics and Use of Space Tethers” supported by the Dirección General de Investigación (DGI) of the Spanish Ministry of Education and Science through contract ESP2007-64068.

References

[1] Bombardelli, C., and Pelaez, J., On the Thrust Capability of Electrodynamic Tethers working in Generator Mode, Proceedings of the 2009 AAS/AIAA Space Flight Mechanics Meeting Conference, Savannah, GA., 2009.

[2] Drell, S. D. and Foley, H. M. and Ruderman, M. A., Drag and Propulsion of Large Satellites in the Ionosphere: An Alfven Propulsion Engine in Space, Journal of Geophysical Research, July 1965, Vol 70, No 13, pp.3131-3145.

[3] Sanmartin, J.R. and Ahedo, E. and MartInez-Sanchez, M., An Anodeless Tether Generator, Proceedings of Physics of Charged Bodies in Space Plasmas, 1991, editors M. Dobrowolny and E. Sindoni.

[4] Sanmartin, J. R., and Lorenzini, E. C., Exploration of Outer Planets Using Tethers for Power and Propulsion, Journal of Propulsion and Power, 2005, vol. 21, pp. 573-576.

[5] C. Bombardelli, M. Sanjurjo and J. Peláez, An asymptotic solution for the current profile of bare electrodynamic tethers working in generator mode, Journal of Propulsion and Power, submitted.